

## A1. Determining the Density of Solids and Liquids Using a Hydrostatic Balance

*The aim of the exercise is to understand the fundamental laws of hydrostatics, the concept of density, and its dependence on temperature, ultimately – experimentally determining the density of various solids and liquids using Archimedes' principle.*

**The density** (or *mass density*) is given by the formula:

$$\rho = \frac{m}{V} \quad (1)$$

where  $m$  is the mass and  $V$  is the volume.

The density is expressed as a quantity of mass contained in a volume (unit) and given in  $\text{kg/m}^3$ .

Due to thermal expansion, the volume of substances changes with increasing temperature (usually increasing) according to the equation:

$$V = V_0(1 + \gamma\Delta t) \quad (2)$$

where  $\gamma$  is the coefficient of volumetric expansion and  $\Delta t$  is the temperature change. Consequently, density decreases with increasing temperature as:

$$\rho = \frac{\rho_0}{1 + \gamma\Delta t} \quad (3)$$

**A hydrostatic balance** allows the measurement of the density of solids (especially those with irregular shapes) based on Archimedes' principle. It can be a standard analytical balance mounted on a special stand, enabling objects to be weighed while suspended below the pan.

**To determine the density of a solid** (with a density greater than water density) the following steps are necessary:

1. Suspend the sample on a thin wire or thread (with negligible mass) and weigh it in the air (let's denote the result as  $m$ ).
2. Weigh the same sample while fully submerged in a reference liquid, usually distilled water (let's denote this result as  $m_1$ ).

Since the weight in liquid is reduced by the buoyant force (opposite to gravity), Archimedes' principle states:

$$F_b = \rho_l V g \quad (4)$$

where  $V$  is the volume of the submerged object (equal to the volume of displaced liquid),  $\rho_l$  is the density of the liquid, and  $g$  is the gravitational acceleration.

The ratio of the object's weight in the air ( $Q = mg = V\rho g$ , where  $\rho$  is the density of the object) to the buoyant force  $F_b$  (see equation no. 4) allows to determine the dimensionless quantity  $A$ , which represents the ratio of the object's density to that of the liquid:

$$A = \frac{Q}{F_b} = \frac{\rho g V}{\rho_l g V} = \frac{\rho}{\rho_l} \quad (5)$$

Since  $F_b = Q - Q_1$ , we get:

$$A = \frac{m}{m - m_1} \quad (6)$$

And finally, the density of the object can be calculated as:

$$\rho = A \rho_l \quad (7)$$

**To determine an unknown liquid density  $\rho_x$ :**

1. Weigh a known solid in the air ( $m$ ).
2. Weigh it fully submerged in a reference liquid of known density – usually distilled water  $\rho_l$  ( $m_1$ )
3. Weigh it fully submerged in the unknown liquid ( $m_2$ )

With the above results, one can determine the dimensionless quantity  $B$ , which represents the ratio of buoyancy forces in both liquids ( $Q-Q_2$  in an unknown liquid related to  $Q-Q_1$  in a reference):

$$B = \frac{Q - Q_2}{Q - Q_1} = \frac{m - m_2}{m - m_1} = \frac{\rho_x}{\rho_l} \quad (8)$$

And finally, the unknown liquid's density:

$$\rho_x = B \rho_l \quad (9)$$

**A1. Measurement and calculation protocol**

Pair No.:	Student's name and surname:	Field of study: Group:
Date:	Teacher's name and surname:	Passing information:

**Measuring protocol:**

**Equipment:** scale/balance, stand, beaker, distilled water, alcohol, copper sulfate solution, salt solution, metal and polymer solid samples

**Precautions:**

- Ensure complete immersion and free suspension of the object without touching the beaker's walls or bottom.
- Rinse and dry the object before switching liquids.

**I. Determining the Density of a Solid:**

1. Weigh the solid in the air ( $m$ ).
2. Weigh the solid fully submerged in water ( $m_1$ ).
3. Repeat for several (two) other solids.
4. Record the results in Table 1.
5. Denote the uncertainty of the mass

**Table 1**

$t = \dots\dots\dots [^{\circ}\text{C}]$

$\rho_w = \dots\dots\dots [\text{kg}/\text{m}^3]$

Sample (unknown)	"mass" of the sample in the air $m$ [g]	"mass" of the sample in distilled water $m_1$ [g]	A	$\Delta A$	Sample density $\rho$ [ $\text{kg}/\text{m}^3$ ]	$\Delta\rho$ [ $\text{kg}/\text{m}^3$ ]

$\Delta m = \Delta m_1 = \dots\dots\dots$  [g]

**Data analysis and calculations:**

1. For each sample calculate A using  $A = \frac{m}{m - m_1}$
2. Determine  $\rho$  using  $\rho = A \rho_w$  (the density of the distilled water at a given temperature can be found in the table available in the class)
3. Calculate the uncertainty of A using standard differentiation (for each sample, separately):

$$\Delta A = \left| \frac{-m_1}{(m - m_1)^2} \right| \Delta m + \left| \frac{-m}{(m - m_1)^2} \right| \Delta m_1$$

and the uncertainty of  $\rho_x$ , using the logarithmic method (for each sample separately):

4. Finally present the result for each sample as  $(\rho \pm \Delta\rho)$  in SI units

$$\frac{\Delta\rho}{\rho} = \frac{\Delta A}{A}, \text{ therefore: } \Delta\rho = \rho \left| \frac{\Delta A}{A} \right|$$

## II. Determining the Density of a Liquid:

1. Choose at least three random (unknown) solid samples (you can take the same three samples, as for the first part of the protocol). Those are called auxiliary (reference solids), further noted as solids.
2. Weigh the first reference solid in the air ( $m$ ).
3. Weigh the solid fully submerged in water ( $m_1$ ).
4. Weigh the solid fully submerged in the test liquid ( $m_2$ ).
5. Repeat steps 2-4 with different reference solids for the same liquid
6. Record the results in Table 2.

**Table 2**

tested liquid:.....

Reference solid (No.)	“mass” of the sample in the air $m$ [g]	“mass” of the sample in distilled water $m_1$ [g]	“mass” of the sample in the test liquid $m_2$ [g]	B	The density of the tested liquid $\rho_x$ [kg/m <sup>3</sup> ]
Mean $\rho_x =$					
The uncertainty of mean $\Delta\rho_x =$					
Final result (rounded) $(\rho_x \pm \Delta\rho_x) =$					

### Data analysis and calculations:

1. For each sample calculate  $B$  using
2. determine  $\rho$  using  $\rho_x = B \rho_w$  (the density of the distilled water at a given temperature can be found in the table available in the class)
3. Calculate the mean density of the liquid and its uncertainty as a tripled standard deviation of the mean:

$$\Delta\rho_x = 3 SD = 3 \sqrt{\frac{\sum_{i=1}^n (\rho_i - \bar{\rho}_x)^2}{n(n-1)}}$$

4. Finally present the result for the sample as  $(\rho_x \pm \Delta\rho_x)$  in SI units.