C12. Determining the Temperature Dependence of the Viscosity Coefficient of Liquids

***The aim of the exercise** is to understand the phenomenon of liquid viscosity, its changes as a function of temperature, its effect on the dynamics of object motion in viscous media, and the experimental determination of the activation energy of viscous flow by measuring the temperature dependence of the viscosity coefficient.*

The viscosity of the liquid, also known as **internal friction**, is the resistance force caused by intermolecular interactions between two adjacent layers of liquid moving relative to each other. This force is tangential to the surface of the layers and is directly proportional to both the **surface area** (*S*) of the considered layer and the **velocity gradient** *dv/dx* (i.e., the change in velocity over a distance between layers). Newton’s equation describes this relationship:

$$F = \eta\, S\, \frac{dv}{dx} \tag{1}$$

where h is the **viscosity coefficient**, which is dependent on the substance and temperature.

According to the kinetic-molecular theory, viscosity arises due to the transfer of momentum between layers that are moving relative to each other. As molecules move from one layer to another, they carry momentum, causing the faster layer to accelerate the slower one and vice versa. To observe viscosity, liquid motion must be induced, such as fluid flow or an object movement within the liquid (causing internal friction between the molecules adhering to the object and the surrounding liquid).
The second case can be considered as a sphere of radius ***r*** moving through a viscous liquid under the influence of an external force. The **viscous force** acting on the sphere is described by **Stokes's law**:

$$F = 6\,\pi\,\eta\,r\,v \tag{2}$$

where v is the velocity of the sphere.
By measuring the falling velocity of a sphere in a liquid, we can determine the liquid's **viscosity coefficient** $\boldsymbol{\eta}$. This principle is used in the **Höppler viscometer**, which will be used in this experiment.

Considering the forces acting on the sphere falling in the viscous liquid, the following can be defined:

- ✓ **Gravitational force, acting downwards:**

$$Q = mg = \rho_s V_s g = \rho_s \frac{4}{3}\pi r^3 g \tag{3}$$

where ρ_s is the density of the sphere and V_s is the volume of the sphere.

- ✓ **Buoyant force** – opposite to gravity force. According to Archimedes’ principle:

$$B = m_l g = \rho_l \frac{4}{3}\pi r^3 g \tag{4}$$

where ρ_s is the liquid density.

- ✓ **Viscous (resistant) force** – defined by the Stokes law (equation 2).

Describing the motion of the sphere, one can observe that, initially, the sphere **accelerates**, but as its velocity increases, the **viscous force increases** until it balances the combined effect of gravity and buoyancy. Eventually, the equilibrium is reached when the net force is 0:

$$Q = F + B \tag{5}$$

According to **Newton's First Law**, the sphere then falls at **constant terminal velocity** $\boldsymbol{v}$. From equation (5), one can note:

$$\rho_s \frac{4}{3}\pi r^3 g = 6\pi\eta r v + \rho_l \frac{4}{3}\pi r^3 g. \tag{6}$$

What leads to the expression for **the viscosity coefficient** $\boldsymbol{\eta}$:

$$\eta = \frac{2r^2 g(\rho_s - \rho_l)}{9v} \tag{7}$$

In the experiment, the velocity of the sphere is assesed by the measurement of time $\boldsymbol{\tau}$ it takes for the sphere to fall a **distance** $\boldsymbol{l}$ between marked levels in the viscometer $\left(v = \frac{l}{\tau}\right)$. Considering this and taking into account that, the expression $\frac{2r^2 g}{9l}$ is constant for a given viscometer and can be defined as **a viscometer constant** $\boldsymbol{K}$*****, the equation (7) can be rewritten in a simplified expression:

$$\eta = K(\rho_s - \rho_l)\tau \tag{8}$$

> *The K constant can be determined experimentally, based on measurements described above taken for a liquid of a known viscosity, but in most cases, its value is provided by the manufacturer. The constant provided also includes a correction arising from a low ratio of sphere to cylinder radius (causing an increase in viscous force)

Viscosity **decreases exponentially** with increasing temperature, described by the **Arrhenius equation**:

$$\eta = Ce^{\frac{E}{RT}} \tag{9}$$

where: C is a constant, E is the **activation energy of viscous flow***, R is the gas constant**,** T is the absolute temperature (Kelvin).

> *The **activation energy** (E) represents the **energy barrier** that molecules must overcome to move relative to each other. It can be experimentally determined by studying how **viscosity changes with temperature**—as done in this experiment.

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Pair No.:	Student's name and surname:	Field of study: Group:
Date:	Teacher's name and surname:	Passing information:

Measuring protocol:

Equipment: Höppler viscometer with ultrathermostat, stopwatch.

Measuring steps:

1. Record the liquid temperature t_1 from the thermometer in the thermostat. The thermostatic water surrounds the cylinder with examined fluid. Therefore, the temperature of the water is the same as the temperature of the fluid measured.
2. Measure the fall time τ of the sphere three times between the levels indicated in the viscometer cylinder.
3. Turn on the ultrathermostat and increase the liquid temperature by about 5°C using the knob of contact thermometer. The reach of the desired temperature is indicated by the turn-off the indicator light.
4. Wait 1-2 minutes for temperature stabilization, then repeat the time measurements.
5. Continue increasing the temperature in 5°C increments and repeat fall time of the sphere until reaching ~50°C.

Data table

L.p.	Temperature			Fall time of the sphere [s]					
	t [°C]	T [K]	1/T [1/K]	τ_1	τ_2	τ_3	$\tau_{average}$	η [Nsm^{-2}]	ln(η)

Data analysis and calculations:

1. Calculate the average fall time $\tau_{average}$ for each temperature.
2. Calculate the viscosity coefficient η using equation (8), where:
   - K=5.42×10^{-5} [N·m/kg]
   - ρ_s=2409 [kg/m³] (density of the sphere)
   - ρ_l=800 [kg/m³] (density of the liquid)
3. Calculate $ln(\eta)$ and convert temperatures to Kelvin and then the reverse of the temperature (1/T).
4. Plot $ln(\eta)$ vs. $1/T$ on graph paper and determine the slope A.
5. Calculate the activation energy, according to the following equation*:

$$E = A\,R \tag{10}$$

where R=8.31 [J/(mol K)]

*<u>explanation:</u> the formula after logarithmization takes form of: $ln(\eta) = \ln(C) + \frac{E}{R}\frac{1}{T}$, which is the equation of a straight line of type $y=B+Ax$, where $y=ln(\eta)$, $x=1/T$, and the slope $A=E/R$. Therefore finding the slope allows the determining the activation energy easily ($E=A\,R$).
The calculation of the slope value and its uncertainty can be performed with the use of the software.

After finding the lope A and its uncertainty ΔA, note the rounded values with correct units:

(A ± ΔA) = (……………………….. ± …………………….) […………]

6. Determine the uncertainty ΔE by the logarithmic method:

$$\Delta E = E\left|\frac{\Delta A}{A}\right| \tag{10}$$

7. Present the final result**:**

(E ± ΔE) = (……………………….. ± …………………….) […………]