MEASUREMENT ERRORS

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CLASSIFICATION OF MEASUREMENTS OF PHYSICAL QUANTITIES

SIMPLE MEASUREMENT

A measurement in which the final result **x** is read directly from the measuring instrument.

Examples of such measurements include:

- Measuring length using a ruler, caliper, or micrometer,
- Reading current intensity from an ammeter,
- Measuring time with a stopwatch,
- Reading temperature from a thermometer.

A quantity obtained through a simple measurement is called a **simple physical quantity**.

COMPLEX MEASUREMENT

A measurement in which the numerical value of the measured physical quantity **W** is obtained only after substituting the results of previously conducted measurements of several independent simple quantities (e.g., x_1, x_2, x_3) into the appropriate formula and performing calculations:

$$W = W(x_1, x_2, x_3)$$

A simple example of such a measurement is the determination of the volume **V** of a rectangular cuboid by:

a) Measuring the three different edge lengths x_1, x_2, x_3 of the prism,
b) Substituting the obtained results into the formula and calculating the volume:

$$V = V(x_1, x_2, x_3) = x_1 x_2 x_3$$

The quantity obtained through a complex measurement is called a **complex physical quantity**.

CLASSIFICATION AND CALCULATION OF MEASUREMENT ERRORS FOR SIMPLE PHYSICAL QUANTITIES

SYSTEMATIC ERRORS

These errors result from the accuracy class of the measuring instrument and are made with each reading. If **Δx** represents the smallest scale division of the measuring instrument used for a simple quantity **x**, then one can say that:

- The measurement **x** is repeatable with an accuracy of **Δx**,
- The measurement is subject to an error **Δx**.

The recorded measurement result should include the associated error:

x=(x±Δx) [units]

RANDOM ERRORS

These errors manifest as variations in measurement values during repeated readings. In such cases, a series of **n** measurements of the same physical quantity **x** should be conducted, and the arithmetic mean is taken as the final result.

The error of the mean value **Δx̄** for a small series of **n** repeated measurements is calculated using the standard deviation of mean value formula:

$$\Delta\bar{x} = 3\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n(n-1)}} = 3\sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n(n-1)}}$$

The scientific calculator in SD (Standard Deviation) mode calculates the standard deviation according to one of the following two formulas:

$$\sigma_n = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} \quad \text{or} \quad \sigma_{n-1} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Therefore, in the case of calculations performed using a calculator, the value of the error which is burdened with the average value of **n** measurements is calculated using the expressions:

$$\Delta\bar{x} = 3\frac{\sigma_n}{\sqrt{n-1}} \text{ or } \Delta\bar{x} = 3\frac{\sigma_{n-1}}{\sqrt{n}}$$

CALCULATION OF MEASUREMENT ERRORS FOR COMPLEX PHYSICAL QUANTITIES

MAXIMUM SYSTEMATIC ERROR OF A COMPLEX PHYSICAL QUANTITY

To determine the maximum error **ΔW** of a physical quantity **W**, calculated based on the measurements of several simple quantities **x_1, x_2, x_3** (each with its respective error **Δx_1, Δx_2, Δx_3**), two methods can be used:

a) **<u>Total Differential Method</u>**

This method applies to any function **$W = W(x_1, x_2, x_3)$**, regardless of its analytical form:

$$\Delta W = \left|\frac{\partial W}{\partial x_1}\right| \Delta x_1 + \left|\frac{\partial W}{\partial x_2}\right| \Delta x_2 + \left|\frac{\partial W}{\partial x_3}\right| \Delta x_3$$

b) **<u>The logarithmic method</u>**

This method is applicable only when **W** is expressed as a product of a constant **C** and arbitrary powers of the measured quantities **x_1, x_2, x_3**:

$$W = C x_1^n x_2^m x_3^k \quad ,$$

where **C** is a numerical constant, and **n, m, k** are rational numbers. In this case, the maximum error is given by:

$$\frac{\Delta W}{W} = \left|n \frac{\Delta x_1}{x_1}\right| + \left|m \frac{\Delta x_2}{x_2}\right| + \left|k \frac{\Delta x_3}{x_3}\right|$$

⚠ **Note:** The individual components in the formulas represent the partial errors contributed by each simple measurement. Absolute values prevent negative errors from affecting the final error calculation.

ROUNDING RULES FOR MEASUREMENT RESULTS AND ERRORS

Before presenting the final measurement result **W** along with its error **ΔW**, appropriate rounding procedures must be applied using the following rules:

1. First, round the error **ΔW**.
2. Express **ΔW** in the form:

$$\Delta W = 0.abc \times 10^{n}$$

3. If **a > 3**, round **ΔW** to one significant digit.
4. If **a ≤ 3**, round **ΔW** to two significant digits.
5. Express the measured quantity **W** in a similar decimal notation:

$$W = A.BCD \times 10^{n}$$

6. Round **W** to match the decimal place of the rounded **ΔW**.
7. The decision to round up or down follows standard mathematical rounding rules.
8. The final measurement result should be presented in scientific notation as:

$$W = (W \pm \Delta W) \times 10^{n} \text{ [units]}$$

⚠ **Note:** The exponent **n** in the final presentation may be adjusted to align with SI unit conventions or standard representations for the given physical quantity.