Reviving pseudo-inverses: Asymptotic properties of large dimensional Moore-Penrose and ridge-type inverses with applications

Taras Bodnar[1], **Nestor Parolya**[2]

[1]*Department of Mathematics, Stockholm University, Stockholm, Sweden*
[2]*Department of Applied Mathematics, Delft University of Technology, Delft, The Netherlands*

Abstract

In this paper, we derive high-dimensional asymptotic properties of the Moore-Penrose inverse and the ridge-type inverse of the sample covariance matrix (see, e.g., [1], [2]). In particular, the analytical expressions of the weighted sample trace moments are deduced for both generalized inverse matrices and are present by using the partial exponential Bell polynomials which can easily be computed in practice. The existent results are extended in several directions: (i) First, the population covariance matrix is not assumed to be a multiplier of the identity matrix; (ii) Second, the assumption of normality is not used in the derivation; (iii) Third, the asymptotic results are derived under the high-dimensional asymptotic regime. Our findings are used in the construction of improved shrinkage estimators of the precision matrix that minimizes the Frobenius norm. Also, shrinkage estimators for the coefficients of the high-dimensional regression model and the weights of the global minimum variance portfolio are obtained. Finally, the finite sample properties of the derived theoretical results are investigated via an extensive simulation study.

Keywords

Moore-Penrose inverse, Ridge-type inverse, Bell polynomials, Sample covariance matrix, Random matrix theory, High-dimensional asymptotics

References

[1] Bodnar, T., H. Dette and N. Parolya (2016). Spectral analysis of the Moore-Penrose inverse of a large dimensional sample covariance matrix. *Journal of Multivariate Analysis 148*, 160–172.

[2] Imori, S. and D. von Rosen (2020). On the mean and dispersion of the Moore-Penrose generalized inverse of a Wishart matrix. *The Electronic Journal of Linear Algebra 36*, 124–133.