Invariant quadratic forms in time series kriging: Distribution structure & computation

Andrej Gajdoš[1], **Martina Hančová**[1], **Jozef Hanč**[2]

[1]*Institute of Mathematics, P. J. Šafárik University, Košice, Slovakia*
[2]*Institute of Physics, P. J. Šafárik University, Košice, Slovakia*

Abstract

Computational research, particularly in time series prediction using linear mixed regression models [1], calls for estimators of variance parameters with optimal statistical properties, minimal computational complexity, and non-iterative computation methods. One such suitable candidate, particularly effective for Monte Carlo and bootstrap simulations, is an estimator in the form of a linear combination of invariant quadratic forms (IQFs). These estimators naturally arise from the non-iterative method of moments and have an ideal computational complexity of $\mathcal{O}(n)$.

Concerning the distribution structure of such a linear combination, we derived a theorem for IQF distributions — a necessary and sufficient condition to achieve a Gamma Difference Distribution ($\mathcal{GDD}$). This result extends the standard distributional theorems for quadratic forms [2]. For computing the $\mathcal{GDD}$ distribution [3], we adopted the numerical inversion of the characteristic function through double exponential quadrature, enabling the very fast and accurate calculation of such distributions [4, 5]. To validate the properties of the derived algorithms, we also implemented a comprehensive simulation study based on a systematic design of simulation experiments [6].

The implications of our results extend beyond econometrics, tackling measurement problems in natural sciences and educational data. Furthermore, it contributes to the development of algorithms for non-negative estimators based on least squares or maximum likelihood [1], broadening the range of applications for our time series forecasting approach.

Keywords

Time series prediction, Computational research, Invariant quadratic forms, Gamma difference distribution, Systematic design of experiments

Acknowledgements

This work was supported by the Slovak Research and Development Agency under the Contract no. APVV-21-0216 and APVV-21-0369.

References

[1] Hančová, M., Gajdoš, A., Hanč, J., and Vozáriková, G. (2021). Estimating variances in time series kriging using convex optimization and empirical BLUPs. *Stat Papers 62*(4), 1899–1938.

[2] Searle, S. R., and Khuri, A. I. (2017). *Matrix Algebra Useful for Statistics* (2nd ed.). Wiley, Hoboken.

[3] Shorack, G. R. (2017). *Probability for Statisticians* (2nd ed.). Springer, New York.

[4] Witkovsky, V. (2016). Numerical inversion of a characteristic function: An alternative tool to form the probability distribution of output quantity in linear measurement models. *ACTA IMEKO 5*(3), 32–44.

[5] Hančová, M., Gajdoš, A., and Hanč, J. (2022). A practical, effective calculation of gamma difference distributions with open data science tools. *J. Stat. Comput. Simul. 92*(11), 2205–2232.

[6] Hocke, S., Meyer, M., and Lorscheid, I. (2015). Improving simulation model analysis and communication via design of experiment principles: an example from the simulation-based design of cost accounting systems. *J Manag Control 26*(2-3), 131–155.