# Quadratic Shrinkage for Large Covariance Matrices

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**Abstract**

This paper constructs a new estimator for large covariance matrices by drawing a bridge between the classic [1] estimator in finite samples and recent progress under large-dimensional asymptotics. The estimator keeps the eigenvectors of the sample covariance matrix and applies shrinkage to the *inverse* sample eigenvalues. The corresponding formula is *quadratic*: it has two shrinkage targets weighted by quadratic functions of the concentration (that is, matrix dimension divided by sample size). The first target dominates mid-level concentrations and the second one higher levels. This extra degree of freedom enables us to outperform linear shrinkage when optimal shrinkage is not linear (which is the general case). Both of our targets are based on what we term the "Stein shrinker", a local attraction operator that pulls sample covariance matrix eigenvalues towards their nearest neighbors, but whose force diminishes with distance, like gravitation. We prove that no cubic or higher-order nonlinearities beat quadratic with respect to Frobenius loss under large-dimensional asymptotics. Non-normality and the case where the matrix dimension exceeds the sample size are accommodated. Monte Carlo simulations confirm state-of-the-art performance in terms of accuracy, speed, and scalability.

**Inverse shrinkage, Hilbert transform, large-dimensional asymptotics, signal amplitude, Stein shrinkage.**

## References

[1] Stein, C. (1975). Estimation of a covariance matrix. Rietz lecture, 39th Annual Meeting IMS. Atlanta, Georgia.