Singular inverse Wishart distribution and its application to portfolio theory

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Abstract

The inverse of the standard estimate of covariance matrix is frequently used in the portfolio theory to estimate the optimal portfolio weights and related characteristics. For this problem, the distribution of the linear transformation of the inverse is needed. We obtain this distribution in the case when the sample size is smaller than the dimension, the underlying covariance matrix is singular, and the vectors of returns are independent and normally distributed. For the result, the distribution of the inverse of covariance estimate is needed and it is derived and referred to as the singular inverse Wishart distribution. We use these results to provide very useful stochastic representations for the characteristics of the expected utility optimal portfolio. Using these stochastic representations, we derive the moments of higher order of the estimated expected return and the estimated variance of the expected utility optimal portfolio. Another line of applications leads to their asymptotic distributions obtained in the high-dimensional setting. Via a simulation study, it is shown that the derived high-dimensional asymptotic distributions provide good approximations of the exact ones even for moderate sample sizes.

The talk is based on papers [1] and [2].

Keywords

Singular Wishart distribution, Mean-variance portfolio, Sample estimate of precision matrix, Moore-Penrose inverse.

References

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[2] Bodnar, T., Mazur, S. and H. Nguyen (2022). Estimation of optimal portfolio compositions for small sample and singular covariance matrix. *Working paper 15*, Örebro University School of Business.