Logarithmic law of large random correlation matrices

Nestor Parolya[1], **Johannes Heiny**[2], **Dorota Kurowicka**[1]

[1] *Delft Institute of Applied Mathematics, TU Delft, The Netherlands*
[2] *Department of Mathematical Statistics, Stockholm University, Sweden*

Abstract

Consider a random vector $\mathbf{y} = \mathbf{\Sigma}^{1/2}\mathbf{x}$, where the p elements of the vector $\mathbf{x}$ are i.i.d. real-valued random variables with zero mean and finite fourth moment, and $\mathbf{\Sigma}^{1/2}$ is a deterministic $p \times p$ matrix such that the eigenvalues of the population correlation matrix $\mathbf{R}$ of $\mathbf{y}$ are uniformly bounded away from zero and infinity. In this paper, we find that the log determinant of the sample correlation matrix $\hat{\mathbf{R}}$ based on a sample of size n from the distribution of $\mathbf{y}$ satisfies a CLT (central limit theorem) for $p/n \to \gamma \in (0, 1]$ and $p \leq n$. Explicit formulas for the asymptotic mean and variance are provided. In case the mean of $\mathbf{y}$ is unknown, we show that after re-centering by the empirical mean the obtained CLT holds with a shift in the asymptotic mean. This result is of independent interest in both large dimensional random matrix theory and high-dimensional statistical literature of large sample correlation matrices for non-normal data. Finally, the obtained findings are applied for testing of uncorrelatedness of p random variables. Surprisingly, in the null case $\mathbf{R} = \mathbf{I}$, the test statistic becomes distribution-free and we show analytically that the obtained CLT also holds if the moments of order four do not exist at all, which conjectures a promising and robust test statistic for heavy-tailed high-dimensional data.

This talk is based on papers [1] and [2].

Keywords

Random Matrix Theory, Sample Correlation Matrix, Logarithmic Determinant, Central Limit Theorems, Large Dimensional Asymptotics.

References

[1] Logarithmic law of large random correlation matrices, N. Parolya, J. Heiny and D. Kurowicka, *Bernoulli* 2023. link

[2] Log determinant of large correlation matrices under infinite fourth moment, J. Heiny and N. Parolya, *Ann. Henri Poincaré (B) - Prob. et Stat.* 2023. link