Distance Laplacians of connected graphs

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Abstract

Consider a connected graph G with n vertices. Then the combinatorial Laplacian of G is the $n \times n$ matrix

$$L(G) := \mathrm{Deg}(G) - A(G),$$

where $\mathrm{Deg}(G)$ is the diagonal matrix with degrees on the diagonal and $A(G)$ is the adjacency matrix. $L(G)$ has certain properties: It is an M-matrix, $\mathrm{rank}(L(G))$ is $n-1$ and the sum of each row and column is zero. In general, the Moore-Penrose inverse of $L(G)$ is not an M-matrix and it is quite rare to find connected graphs for which the Moore-Penrose inverse of $L(G)$ is an M-matrix.

In this presentation, we will discuss an alternative Laplacian matrix, denoted as $T(G)$, which retains all the characteristics of a Laplacian while having the special property that the Moore-Penrose inverse of $T(G)$ is an M-matrix. To construct $T(G)$, we replace the adjacency matrix $A(G)$ by a suitable non-negative matrix.

Keywords

Laplacian matrices, complete graphs, M-matrices, resistance matrices.

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