

Sign patterns of inverse doubly nonnegative matrices and inverse completely positive matrices

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Abstract

A square matrix is doubly nonnegative (DNN) if it is both positive semidefinite and entrywise nonnegative. A matrix is completely positive (CP) if it has the form BB^T , where B is an entrywise nonnegative matrix, not necessarily square. Every CP matrix is DNN, but the converse does not hold for matrices of order larger than 4. For matrices of order 5 or more, checking that a matrix is DNN is quite easy, checking that is CP is (NP-)hard.

We consider the sign patterns of inverses of such matrices. We identify all possible sign patterns of inverse DNN matrices, and the sign patterns of inverse completely positive (CP) matrices. We characterize graphs with the property that inverses of all DNN realizations of the graph share the same $\{+, -, 0\}$ sign pattern. We show that these are also the graphs with this property in the completely positive case. In the DNN case, the first result generalizes a result of [?] regarding the $\{+, -\}$ sign pattern of inverse DNN matrices, where $+$ denotes a nonnegative entry. The second result answers a question on DNN matrices left open there. We also consider the reverse question: which $\{+, -, 0\}$ sign patterns of inverse DNN/CP matrices determine uniquely the graph of their originating DNN/CP matrix. We answer the question in the DNN case, but the CP case is still open.

Keywords

Doubly nonnegative matrix, completely positive matrix, sign pattern matrix

References

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