Matrix Identities Associated with Group Representations

Ryo Tabata

Department of General Education,
National Institute of Technology, Ariake College, Japan

Abstract

An immanant is a matrix function associated with the representation of the symmetric group ([4]). The permanent and determinant are the special cases with the trivial and alternating representations, respectively.

The theory of the symmetric functions has been developed with the background of the representation theory. Schur functions form one of the most important basis for the space of symmetric functions. The product of two Schur functions is described in a combinatorial way, called the Littlewood-Richardson rule. In [1], it is shown that immanants also have expansions in terms of matrix minors in the same rule.

Plethysm is another type of the product of Schur functions introduced by Littlewood [2], which originates from the representation theory of the general linear groups. A classical problem to give a combinatorial description for plethysm coefficient remains open since the 1940s.

In this talk, we design matrix identities parallel to plethysm by modifying immanants with a parameter. There is an important connection with Robinson's approach to the representations corresponding to plethysm in terms of the symmetric groups ([3]). We also present some applications to linear algebra.

Keywords

Generalized Matrix Function, Immanant, Symmetric Function, Plethysm.

References

[1] D.E. Littlewood and A.R. Richardson, Group characters and algebra, *Philosophical Transactions of the Royal Society of London. Series A* 233:99–141, 1934.

[2] D. E. Littlewood, Polynomial concomitants and invariant matrices, J. London Math. Soc. 1(1):49-55 (1936).

1

[3] G. de B. Robinson, On the disjoint product of irreducible representations of the symmetric group, Canad. J. Math. 1(2), 166-175 (1949).

[4] I. Schur, Uber endlicher Gruppen und Hermieschen Formen, *Mathematische Zeitschrift* 1:184–207, 1918.