The Moore-Penrose inverse of a Wishart matrix and its first moments

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Abstract

The Moore-Penrose inverse of a Wishart matrix is often used as a plug-in estimator of a dispersion matrix when the degrees of freedom are smaller than the number of columns/rows. If $\mathbf{X} \sim \mathbf{N_{p,n}}(\mathbf{BC}, \mathbf{\Sigma}, \mathbf{I_n})$ (MANOVA model) the sample covariance equals

$$\mathbf{S} = \mathbf{X}(\mathbf{I_n} - \mathbf{C}^\top(\mathbf{CC}^\top)^- \mathbf{C})\mathbf{X}^\top.$$

The sample covariance matrix can always be factorized as $\mathbf{S} = \mathbf{VV}^\top$, where $\mathbf{V}$ is of size $p \times t$, $p > t$ (singular case). Then, if $\mathbf{S}$ is singular its Moore-Penrose can be written

$$\mathbf{S}^+ = \mathbf{V}(\mathbf{V}^\top\mathbf{V})^{-\mathbf{1}}(\mathbf{V}^\top\mathbf{V})^{-\mathbf{1}}\mathbf{V}^\top.$$

Utilizing this result, in the presentation some results for $E[\mathbf{S}^+]$ and $D[\mathbf{S}^+]$ are given.