

Article

Biaxial Inclination of Rectangular Sinking Wells: Analytical Model and Admissible Tilt Envelope

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Abstract

This study presents an analytical framework for evaluating the admissibility of biaxial inclination of rectangular sinking wells. The inclination of the well is interpreted as an eccentric transfer of the vertical load to the concrete plug, which produces a two-dimensional linear stress field beneath the base. Closed-form expressions are derived for the stresses at the four corners of the rectangular base as functions of the eccentricity components associated with the two orthogonal tilt directions. Based on these expressions, the admissibility of inclination is represented by a tilt envelope in the space of the two tilt angles, defining the combinations of tilt components that satisfy the adopted serviceability criterion. The analytical formulation also allows for comparison between the stress-based admissibility limit, the geometric condition corresponding to loss of compressive contact beneath the base, and a simplified indicator of lateral wall-pressure asymmetry acting on the shaft. Parametric analyses show that biaxial inclination leads to stress concentration at the corners of the base and that even relatively small tilt components may combine to produce significant stress amplification. The geometry of the well strongly influences the shape of the admissible tilt envelope, with elongated rectangular wells exhibiting directional anisotropy of the allowable inclination. The proposed analytical approach provides a transparent tool for evaluating inclined wells using basic geometric parameters in engineering practice.

Keywords: biaxial inclination; sinking wells; corner stresses; tilt envelope; eccentric loading; underground structures



Academic Editor: Wei Zhang

Received: 22 March 2026

Revised: 19 April 2026

Accepted: 21 April 2026

Published: 23 April 2026

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1. Introduction

In engineering practice, the design of underground reinforced concrete structures often requires critical decisions under incomplete knowledge of their interaction with surrounding soil [1]. Unlike above-ground structures, which typically face well-defined loads, underground facilities operate within a complex mechanical environment governed by soil behavior [2]. Load transfer depends not only on structural stiffness but also on soil properties, construction methods, and time-dependent processes during both installation and operation [3]. Consequently, analysis of underground structures usually requires a coupled structural and geotechnical approach, treating both systems as an integrated unit [4,5].

This interaction is particularly significant for large monolithic reinforced concrete structures embedded in the ground that directly transfer loads to the subsoil, such as underground water reservoirs, sewage pumping stations, intake chambers, and various hydraulic structures [6]. These systems are generally designed as spatial assemblies of plates and walls interacting with the surrounding soil [7]. Engineering such systems requires a robust verification of stability and contact pressure distribution under varying and non-uniform loading scenarios [8,9]. In this context, deviations from ideal verticality represent one of the most critical and practically relevant issues during construction.

Among construction techniques for underground structures, sinking methods (caissons) are especially advantageous where traditional excavation is difficult or uneconomical [10]. This approach involves gradually lowering a reinforced concrete structure into the ground while excavating soil from its interior [11,12]. It has been widely applied in hydraulic engineering, environmental infrastructure, and bridge foundations because it allows deep installations without extensive temporary support systems [13]. The method is particularly suitable for dense urban areas, where large open excavations are disruptive or impractical [14,15].

From a structural perspective, sinking wells can be considered massive reinforced concrete shells composed of vertical walls, a cutting edge at the base, and, in the final stage, a bottom slab [16]. During sinking, the well initially behaves as a rigid body penetrating the soil under its own weight while internal excavation proceeds [17,18]. In this phase, friction between the walls and surrounding soil generates lateral forces, while the cutting edge at the base experiences concentrated penetration stresses [19]. As the structure descends, these interactions evolve, significantly influencing both the well structural response and the stress distribution in the surrounding soil [20,21].

Once the final depth is reached and the bottom slab is constructed, the structural behavior changes fundamentally [22]. The well becomes a permanent underground facility, functioning either as a foundation element or a storage chamber [23]. At this stage, it behaves similarly to a large reinforced concrete tank, transmitting loads to the supporting soil through the base slab [24,25]. Therefore, the design must account for both transient construction effects and long-term operational loads [26].

Historically, analytical studies have focused on circular sinking wells due to their axisymmetric geometry, which simplifies the analysis of stress and soil pressure [27,28]. Circular wells distribute loads uniformly and avoid geometric discontinuities, making them mechanically favorable [29]. Early engineering practice widely employed circular caissons, which are extensively documented in classical foundation and hydraulic engineering literature [30,31].

However, practical design increasingly favors rectangular wells to accommodate equipment, pipelines, or pumping installations, and to optimize space in constrained urban environments [32]. Rectangular wells present unique structural challenges: unlike circular structures, they exhibit non-uniform load transfer and stress concentrations, particularly at the corners [33,34]. Accurate stress evaluation in rectangular wells requires more advanced analytical methods than those traditionally used for circular caissons [35,36].

Studies on rectangular reinforced concrete tanks and similar spatial plate systems have highlighted the significant influence of geometry on internal force distribution [37]. Parameters such as wall stiffness, rib spacing, and overall proportions strongly affect mechanical performance [38,39]. Numerical simulations and optimization techniques can be used to design structural configurations that maximize load capacity while minimizing material usage [40]. This is especially important for large monolithic structures, where even small geometric changes can substantially alter internal forces and overall material efficiency.

A critical issue during construction is maintaining vertical alignment. Ideally, the structure should descend perfectly vertically, maintaining uniform contact stresses at the base. In practice, soil stratification, local strength variations, and excavation irregularities often cause deviations from the intended axis. Even small tilts can produce eccentric resultant loads on the base, leading to non-uniform soil contact. Classical soil mechanics shows that if the resultant load shifts beyond the foundation's central core, portions of the base may lose contact, causing severe stress concentrations in both the structure and the surrounding soil.

While most analyses consider tilts in a single direction, deviations frequently occur along two perpendicular axes, producing spatial or biaxial inclination. This complicates stress distribution within both the structure and the soil, making simplified one-dimensional analytical models insufficient for a reliable assessment. Biaxial inclination introduces a state of spatial eccentricity that significantly alters internal forces within the walls and the base slab compared to an ideal vertical descent. This shift in the pressure center increases the risk of local soil yielding and imposes complex bending and torsional moments on the reinforced concrete sections. Understanding the limits of such deviations is crucial for establishing safe operational tolerances during sinking. Without a precise analytical framework to quantify these effects, engineers may rely on overly conservative safety factors, which can lead to inefficient material use or, conversely, underestimate the risk of cracking under uneven settlement. Recent studies have increasingly focused on the behavior and analysis of caisson foundations under complex loading and construction conditions. Machine learning-based approaches have been proposed for predicting caisson inclination during the sinking process [34], while monitoring-based analyses have provided insight into structural response during large-scale caisson construction [32]. Probabilistic methods have also been introduced to assess the stability of caisson foundations in spatially variable soils [31]. In addition, the distribution of sidewall earth pressure has been investigated with consideration of soil arching effects, highlighting the importance of soil–structure interaction [40]. Advanced three-dimensional models have further been developed to analyze soil–structure interaction under seismic conditions [30]. Despite these advances, the available approaches remain either limited to one-dimensional eccentric loading or rely on computationally demanding numerical models. Classical eccentric footing theory provides a linear stress distribution under combined axial load and bending, but it is typically applied in a single-axis context and does not directly provide a framework for simultaneous biaxial inclination.

In contrast, the present study extends the classical formulation to a fully biaxial setting, where two independent eccentricity components generate a two-dimensional stress field. This leads to (i) closed-form expressions for corner stresses under combined biaxial eccentricity, (ii) a two-parameter admissibility domain defined in the space of tilt components, and (iii) a unified analytical framework that combines base stress limits, contact-loss conditions, and a simplified indicator of lateral response.

Such a formulation enables direct engineering interpretation of biaxial tilt effects without resorting to numerical simulations, while preserving transparency and computational efficiency. Recent developments in data-driven and monitoring-based approaches further highlight the need for interpretable analytical tools that can complement advanced numerical models in practical applications.

Although modern finite element tools allow detailed soil–structure interaction modeling, their practical use in everyday engineering is limited by computational cost and the expertise required for accurate calibration. Simplified analytical methods therefore remain essential for preliminary design, rapid safety assessment, and on-site decision-making. This study investigates the effect of biaxial inclination on stress distribution at the base of

rectangular sinking wells. It focuses on identifying admissible tilt angles that ensure safe contact with the supporting soil. The proposed analytical framework provides a practical tool for evaluating structural response under spatial inclination, supporting safe sinking operations and informed decision-making during both the design and construction phases. The novelty of the proposed approach lies in (i) the closed-form derivation of corner stress expressions under biaxial eccentricity, (ii) the introduction of a tilt envelope in the angle space, and (iii) the incorporation of a simplified lateral response indicator within a unified admissibility framework.

2. Materials and Methods

2.1. General Assumptions and Geometric Description

The analyzed structure is a reinforced concrete sinking well with a rectangular plan geometry, characterized by the dimensions: B —width of the well, L —length of the well, H —shaft height, t —wall thickness, h_c —thickness of the concrete bottom plug, as shown in Figure 1.

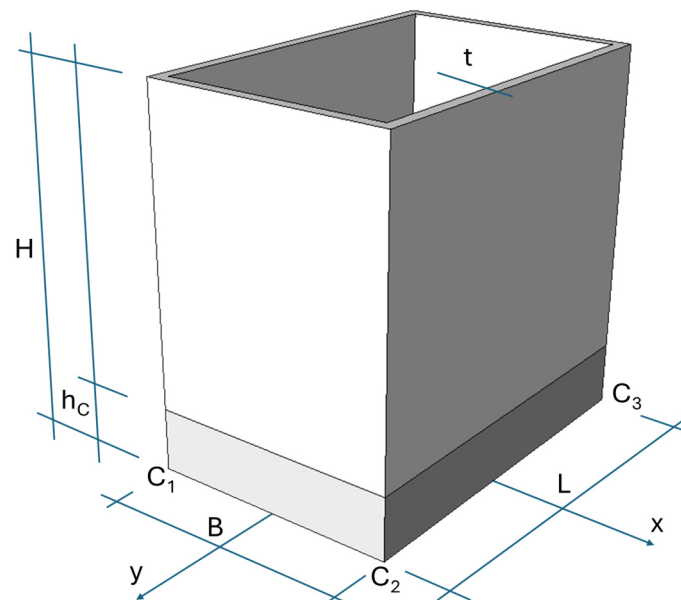


Figure 1. Geometry of the well—*isometric view with principal dimensions.*

The well is assumed to have reached its final foundation depth, and the concrete plug is assumed to be fully executed and rigidly connected to the shaft. The top surface of the plug is horizontal. The well is analyzed in the post-sinking state, when the structure transfers its permanent load to the surrounding soil through the bottom plug.

The total vertical load acting on the system is represented by the sum of: G_s —self-weight of the reinforced concrete shaft, G_p —weight of the cover slab and additional permanent superstructure or soil surcharge, G_c —self-weight of the concrete plug.

Thus, the total vertical force transmitted to the soil is

$$N = G_s + G_p + G_c. \quad (1)$$

The base area of the rectangular plug is

$$A = BL \quad (2)$$

For the perfectly vertical well, the load is transferred concentrically and the stress beneath the plug is assumed to be uniform:

$$\sigma_0 = \frac{N}{A}. \quad (3)$$

The quantity σ_0 is treated as the reference contact stress corresponding to the ideal vertical configuration. The formulation follows the classical rigid-footing analogy, in which the well–soil interaction is represented through resultant forces and moments acting at the base.

The adopted formulation follows the classical rigid-footing analogy. It should be noted, however, that this assumption is most appropriate for relatively stiff structures founded on soils of moderate to high stiffness. In cases involving highly deformable soils or flexible structural systems, deviations from the linear stress distribution may occur, and the present formulation should be interpreted as a first-order approximation.

2.2. Biaxial Inclination and Equivalent Eccentricities

In practical construction conditions, the well may deviate from verticality not only in one plane, but simultaneously in two orthogonal directions. This may occur due to: (a) local obstacles at or near the cutting edge, (b) soil heterogeneity, (c) asymmetric excavation, (d) local dynamic disturbance, (e) or uneven loading around the shaft.

To describe the spatial deviation of the well axis, two tilt angles are introduced: α_x —inclination component associated with the longitudinal direction, α_y —inclination component associated with the transverse direction.

The biaxial inclination is represented through the equivalent eccentricities of the resultant vertical load with respect to the centroid of the plug. Assuming a rigid-body rotation of the shaft and small tilt angles, the eccentricities may be expressed as

$$e_x = \frac{H}{2} \tan \alpha_x. \quad (4)$$

$$e_y = \frac{H}{2} \tan \alpha_y. \quad (5)$$

This formulation assumes that the resultant load acts at mid-height of the shaft, which is consistent with uniform mass distribution and standard engineering approximation. For small angles, the approximation

$$\tan \alpha_x \approx \alpha_x, \quad \tan \alpha_y \approx \alpha_y, \quad (6)$$

may be used when α_x and α_y are expressed in radians. This approximation is valid for tilt angles typically below 5° , which correspond to practical construction deviations. For larger inclinations, higher-order geometric effects may become non-negligible, and the linearized formulation may underestimate stress redistribution. Therefore, the proposed analytical model is intended primarily for serviceability-level assessment of moderate tilts.

The corresponding bending moments transmitted to the base are

$$M_x = Ne_x, \quad (7)$$

$$M_y = Ne_y. \quad (8)$$

These moments generate a two-dimensional linear variation in contact stress beneath the rectangular plug.

2.3. Geometric Properties of the Rectangular Base

To evaluate the biaxial stress field beneath the plug, the second moments of area of the rectangular base with respect to the centroidal axes are introduced:

$$I_x = \frac{BL^3}{12}, \quad (9)$$

$$I_y = \frac{LB^3}{12}, \quad (10)$$

where I_x corresponds to bending that causes stress variation along the y -direction, I_y corresponds to bending that causes stress variation along the x -direction.

The coordinates are measured from the centroid of the plug, with

$$x \in \left[-\frac{B}{2}, \frac{B}{2}\right], \quad y \in \left[-\frac{L}{2}, \frac{L}{2}\right]. \quad (11)$$

2.4. Contact Stress Field Under Biaxial Eccentric Loading

Following classical beam-on-elastic-foundation and rigid footing theory, the contact stress distribution under combined axial force and bending may be expressed as

$$\sigma(x, y) = \frac{N}{A} + \frac{M_x}{I_x}y + \frac{M_y}{I_y}x. \quad (12)$$

Substituting Equations (2) and (7)–(10) into Equation (12) gives

$$\sigma(x, y) = \frac{N}{BL} + \frac{12Ne_x}{BL^3}y + \frac{12Ne_y}{LB^3}x. \quad (13)$$

Equation (13) defines the full linear stress field over the rectangular contact area. The extreme values of the stress occur at the four corners of the base, where both contributions from biaxial bending reach their maximum absolute values. The adopted coordinate system and the corner points of the rectangular base (C1–C4) used later in the derivation of corner stresses are also indicated in Figure 1.

2.5. Corner Stresses

The coordinate system is defined with its origin at the centroid of the rectangular base, with the x -axis aligned with the width B and the y -axis aligned with the length L . The four corner points correspond to the extreme combinations of these coordinates.

The four corner points of the base are defined as

$$C_1 = \left(\frac{B}{2}, \frac{L}{2}\right), \quad (14)$$

$$C_2 = \left(-\frac{B}{2}, \frac{L}{2}\right), \quad (15)$$

$$C_3 = \left(-\frac{B}{2}, -\frac{L}{2}\right), \quad (16)$$

$$C_4 = \left(\frac{B}{2}, -\frac{L}{2}\right). \quad (17)$$

By substituting these coordinates into Equation (13), the corner stresses are obtained as

$$\sigma_1 = \frac{N}{BL} + \frac{6Ne_x}{BL^2} + \frac{6Ne_y}{LB^2}, \quad (18)$$

$$\sigma_2 = \frac{N}{BL} + \frac{6Ne_x}{BL^2} - \frac{6Ne_y}{LB^2}, \quad (19)$$

$$\sigma_3 = \frac{N}{BL} - \frac{6Ne_x}{BL^2} - \frac{6Ne_y}{LB^2}, \quad (20)$$

$$\sigma_4 = \frac{N}{BL} - \frac{6Ne_x}{BL^2} + \frac{6Ne_y}{LB^2}. \quad (21)$$

The maximum and minimum contact stresses are then defined as

$$\sigma_{max} = \max(\sigma_1, \sigma_2, \sigma_3, \sigma_4), \quad (22)$$

$$\sigma_{min} = \min(\sigma_1, \sigma_2, \sigma_3, \sigma_4). \quad (23)$$

For the case in which both eccentricities are taken as positive with respect to the adopted coordinate system, the most compressed corner is C_1 , and the least compressed corner is C_3 . In this common case, Equations (18) and (20) directly provide σ_{max} and σ_{min} . The corner formulation directly reflects the superposition of biaxial bending effects and provides a convenient basis for identifying the governing stress location.

2.6. Dimensionless Form of the Corner Stresses

To obtain a general formulation independent of the absolute load level, the corner stresses may be normalized by the reference stress σ_0 from Equation (3). Dividing Equations (18)–(21) by $\sigma_0 = N/(BL)$ yields

$$\frac{\sigma_1}{\sigma_0} = 1 + \frac{6e_x}{L} + \frac{6e_y}{B}, \quad (24)$$

$$\frac{\sigma_2}{\sigma_0} = 1 + \frac{6e_x}{L} - \frac{6e_y}{B}, \quad (25)$$

$$\frac{\sigma_3}{\sigma_0} = 1 - \frac{6e_x}{L} - \frac{6e_y}{B}, \quad (26)$$

$$\frac{\sigma_4}{\sigma_0} = 1 - \frac{6e_x}{L} + \frac{6e_y}{B}. \quad (27)$$

Thus, for positive e_x and e_y , the normalized extreme stresses are

$$\frac{\sigma_{max}}{\sigma_0} = 1 + \frac{6e_x}{L} + \frac{6e_y}{B}, \quad (28)$$

$$\frac{\sigma_{min}}{\sigma_0} = 1 - \frac{6e_x}{L} - \frac{6e_y}{B}. \quad (29)$$

These two equations constitute the core of the biaxial analytical model. This normalized form reveals that the stress amplification depends solely on geometric ratios, making the formulation independent of the absolute load level.

2.7. Serviceability-Based Admissibility Criterion

To assess whether a tilted well may remain in service, a stress-based admissibility criterion is introduced. The maximum contact stress beneath the base is limited by

$$\sigma_{max} \leq k_\sigma \sigma_0, \quad (30)$$

where k_σ denotes the allowable stress amplification factor.

Following the rationale established for the simplified analytical framework, the value

$$k_{\sigma} = 1.2. \quad (31)$$

may be adopted as a conservative engineering threshold corresponding to a 20% increase in maximum soil stress relative to the vertical configuration [41]. The value $k_{\sigma} = 1.2$ is adopted as a conservative engineering threshold corresponding to moderate stress amplification within the serviceability regime. Other values may be used depending on design requirements.

Substituting Equation (28) into Equation (30) gives

$$1 + \frac{6e_x}{L} + \frac{6e_y}{B} \leq k_{\sigma}, \quad (32)$$

which may be rearranged into

$$\frac{6e_x}{L} + \frac{6e_y}{B} \leq k_{\sigma} - 1. \quad (33)$$

Equation (33) defines the serviceability admissibility domain in the eccentricity space (e_x, e_y) .

2.8. Tilt Envelope in the Angle Space

Using Equations (4) and (5), the eccentricity-based criterion may be rewritten in terms of the tilt angles:

$$\frac{6}{L} \left(\frac{H}{2} \tan \alpha_x \right) + \frac{6}{B} \left(\frac{H}{2} \tan \alpha_y \right) \leq k_{\sigma} - 1. \quad (34)$$

After simplification,

$$\frac{3H}{L} \tan \alpha_x + \frac{3H}{B} \tan \alpha_y \leq k_{\sigma} - 1. \quad (35)$$

Equation (35) defines the tilt envelope in the (α_x, α_y) space. It represents the family of biaxial tilt combinations that satisfy the adopted serviceability criterion. The resulting inequality defines a linear boundary in the angle space, which represents the admissible combinations of biaxial tilt components.

If $\alpha_y = 0$, Equation (35) reduces to the one-dimensional limit

$$\tan \alpha_{x,\text{lim}} = \frac{(k_{\sigma} - 1)L}{3H}, \quad (36)$$

and therefore

$$\alpha_{x,\text{lim}} = \arctan \left(\frac{(k_{\sigma} - 1)L}{3H} \right). \quad (37)$$

Similarly, for $\alpha_x = 0$,

$$\tan \alpha_{y,\text{lim}} = \frac{(k_{\sigma} - 1)B}{3H}, \quad (38)$$

$$\alpha_{y,\text{lim}} = \arctan \left(\frac{(k_{\sigma} - 1)B}{3H} \right). \quad (39)$$

These expressions correspond to the projections of the biaxial envelope onto the coordinate axes.

2.9. Contact-Loss Criterion and Kern Boundary

A second important limit state corresponds to the loss of compressive contact between the plug and the soil. In the adopted analytical framework, this occurs when the minimum contact stress reaches zero:

$$\sigma_{min} = 0. \quad (40)$$

Using Equation (29), the normalized condition becomes

$$1 - \frac{6e_x}{L} - \frac{6e_y}{B} = 0, \quad (41)$$

or equivalently

$$\frac{6e_x}{L} + \frac{6e_y}{B} = 1. \quad (42)$$

This equation defines the biaxial kern boundary of the rectangular base. In the angle space, the contact-loss condition may be written as

$$\frac{3H}{L} \tan \alpha_x + \frac{3H}{B} \tan \alpha_y = 1. \quad (43)$$

Comparison of Equation (35) with Equation (43) shows that, for $k_\sigma = 1.2$, the serviceability boundary lies substantially inside the geometric limit associated with loss of contact. This confirms that the adopted stress criterion is conservative with respect to the full-contact condition. This condition corresponds to the classical kern of a rectangular footing under eccentric loading.

2.10. Simplified Model of Asymmetric Lateral Earth Pressure

Inclination of the well modifies not only the stress distribution beneath the plug, but also the lateral interaction between the shaft and the surrounding soil. In practice, the side toward which the well tilts may experience increased lateral resistance, while the opposite side becomes relatively unloaded. This mechanism has been discussed qualitatively in earlier analyses of inclined sinking wells.

To capture this effect in a transparent analytical form, a simplified scenario-based model of wall pressure asymmetry is introduced.

The baseline triangular earth pressure distribution is assumed as

$$p_0(z) = K\gamma'z, \quad (44)$$

where K —earth pressure coefficient, γ' —effective unit weight of soil, z —depth below ground level.

For the pair of walls normal to the x -direction, the pressure distributions are written as

$$p_{x-}(z) = K\gamma'z(1 - \lambda_x), \quad (45)$$

$$p_{x+}(z) = K\gamma'z(1 + \lambda_x). \quad (46)$$

Similarly, for the pair of walls normal to the y -direction,

$$p_{y-}(z) = K\gamma'z(1 - \lambda_y), \quad (47)$$

$$p_{y+}(z) = K\gamma'z(1 + \lambda_y). \quad (48)$$

The asymmetry coefficients are assumed proportional to the corresponding tilt components:

$$\lambda_x = c_\lambda \tan \alpha_y, \quad (49)$$

$$\lambda_y = c_\lambda \tan \alpha_x. \quad (50)$$

where c_λ is a model coefficient controlling the sensitivity of the lateral pressure to the tilt angle. The coefficient c_λ is introduced as a phenomenological parameter that controls the sensitivity of lateral pressure redistribution to the tilt angle. Its value is not derived from first principles but reflects the assumed degree of coupling between inclination and soil reaction. In practical applications, c_λ may depend on soil type, wall roughness, and construction conditions, and should therefore be treated as a scenario-dependent parameter rather than a calibrated design constant.

The cross-coupling in Equations (49) and (50) reflects the fact that rotation in one direction modifies the soil reaction on the opposite walls perpendicular to that direction. The proposed formulation is phenomenological and intended to capture first-order trends rather than provide a rigorous geotechnical description of soil–structure interaction. It should therefore be interpreted as a qualitative indicator rather than a design-level prediction. The assumed redistribution of lateral earth pressure acting on the shaft walls is schematically illustrated in Figure 2.

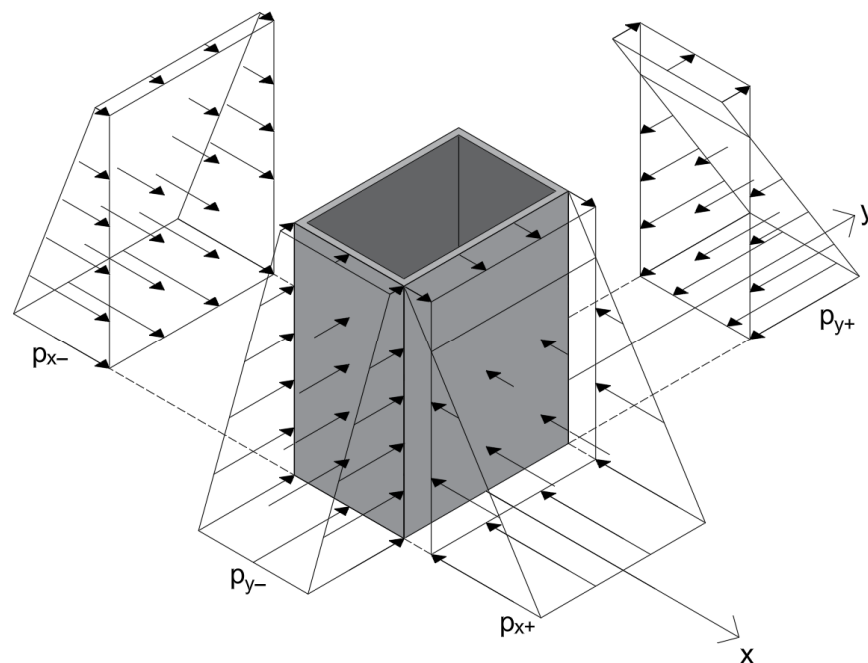


Figure 2. Earth pressure distribution on the well walls.

2.11. Resultant Lateral Forces on the Walls

For a triangular pressure distribution acting on a wall of height H and plan length b , the resultant force is

$$P = \frac{1}{2} K \gamma' H^2 b. \quad (51)$$

Accordingly, for the x -walls, whose plan length equals L , the resultants are

$$P_{x-} = \frac{1}{2} K \gamma' H^2 L (1 - \lambda_x), \quad (52)$$

$$P_{x+} = \frac{1}{2} K \gamma' H^2 L (1 + \lambda_x). \quad (53)$$

For the y -walls, whose plan length equals B ,

$$P_{y-} = \frac{1}{2} K \gamma' H^2 B (1 - \lambda_y), \quad (54)$$

$$P_{y^+} = \frac{1}{2} K \gamma' H^2 B (1 + \lambda_y). \quad (55)$$

The corresponding force ratios are

$$R_{p,x} = \frac{P_{x^+}}{P_{x^-}} = \frac{1 + \lambda_x}{1 - \lambda_x}, \quad (56)$$

$$R_{p,y} = \frac{P_{y^+}}{P_{y^-}} = \frac{1 + \lambda_y}{1 - \lambda_y}. \quad (57)$$

The global wall asymmetry index is defined as

$$R_p = \max(R_{p,x}, R_{p,y}). \quad (58)$$

2.12. Wall-Pressure Admissibility Criterion

To limit excessive asymmetry of lateral action on the shaft, an admissibility condition for the wall response is introduced:

$$R_p \leq R_{p,lim}, \quad (59)$$

where $R_{p,lim}$ is the allowable wall-pressure asymmetry ratio. The parameter $R_{p,lim}$ is introduced as a scenario-dependent threshold reflecting acceptable imbalance of lateral soil reaction.

This condition provides a practical indicator of whether the biaxial tilt causes excessively unbalanced soil action on the shaft, even if the base stress criterion is still satisfied.

The adopted value of $R_{p,lim}$ may be selected depending on the intended conservatism of the analysis. In the present study it is treated as a scenario parameter rather than as a universal code-based limit.

2.13. Combined Admissibility Criterion

The complete analytical framework combines three conditions:

- (i) base stress criterion

$$\sigma_{max} \leq k_\sigma \sigma_0, \quad (60)$$

- (ii) full-contact criterion

$$\sigma_{min} \geq 0, \quad (61)$$

- (ii) wall asymmetry criterion

$$R_p \leq R_{p,lim}. \quad (62)$$

The biaxial tilt state is considered admissible only if all three conditions are simultaneously fulfilled.

Thus, the combined admissibility domain in the (α_x, α_y) space may be written as

$$\mathcal{D} = \left\{ (\alpha_x, \alpha_y) : \sigma_{max} \leq k_\sigma \sigma_0, \sigma_{min} \geq 0, R_p \leq R_{p,lim} \right\}. \quad (63)$$

Equation (63) provides the basis for the graphical construction of the admissible biaxial tilt region presented later in the Section 3. This formulation enables identification of the governing mechanism controlling admissibility for a given tilt configuration.

2.14. Scope and Applicability of the Proposed Model

The proposed analytical framework is based on several simplifying assumptions. The well is assumed to behave as a rigid body, and the concrete plug is considered rigid and fully bonded to the shaft. The stress distribution beneath the base is assumed to remain

linear, while the soil reaction beneath the plug is treated as purely compressive within the admissible range of loading conditions. The lateral earth pressure model adopted in the analysis is simplified and scenario-based, providing an indicative representation of the asymmetry of wall reactions associated with well inclination. In addition, the analysis is performed under static conditions and does not consider time-dependent effects such as consolidation, creep, or cyclic loading. Within these assumptions, the model provides a transparent closed-form tool for evaluating the admissibility of biaxial inclination of rectangular sinking wells while preserving direct engineering interpretability. The simplicity of the formulation is intentional and constitutes a key advantage for rapid engineering assessment.

3. Results

3.1. Reference Configuration and Response Components

The analytical framework derived in Section 2 was first applied to a reference rectangular sinking well in order to illustrate the coupled effects of biaxial tilt on the base reaction and on the lateral soil action. The adopted reference parameters are summarized in Table 1.

Table 1. Reference geometric, material and model parameters adopted in the biaxial tilt analysis of the rectangular sinking well.

Parameter	Symbol	Value
Width	B	4.00 m
Length	L	6.00 m
Shaft height	H	8.00 m
Wall thickness	t	0.25 m
Plug thickness	h_c	1.00 m
Concrete unit weight	γ_c	25.0 kN/m ³
Surface surcharge	q_p	35.0 kN/m ²
Base stress amplification limit	k_σ	1.20
Wall sensitivity coefficient	c_λ	1.00
Wall asymmetry limit	$R_{p,lim}$	1.10

The considered reference case corresponds to a well of moderate dimensions, representative of practical underground infrastructure applications. The adopted tilt components were selected so that the reference state remains close to the serviceability boundary while still preserving full compressive contact beneath the base. The geometric configuration, the biaxial eccentricity vector, and the resulting load transfer scheme are illustrated in Figure 3.

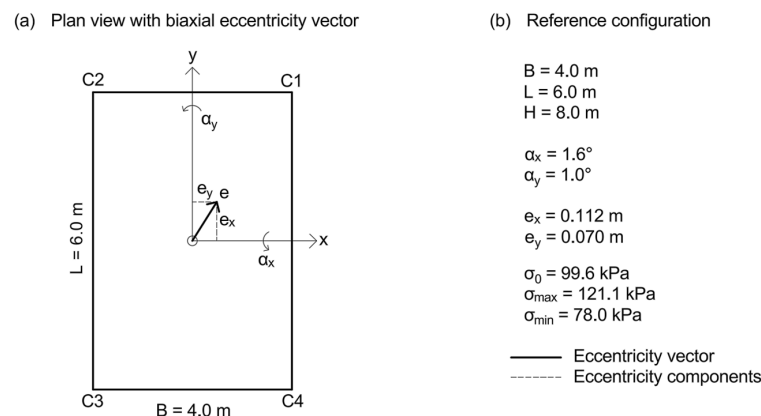


Figure 3. Geometry of the biaxial tilt problem and the reference loading state of the rectangular sinking well: (a) plan view with the eccentricity vector of the resultant vertical load; (b) summary of the reference geometric parameters, tilt angles, and characteristic stress measures.

As shown in Figure 3, the biaxial tilt is conveniently represented by the two eccentricity components e_x and e_y , which generate the bending moments M_x and M_y at the level of the plug-soil contact. This representation provides the basis for the corner-stress evaluation presented below, where the biaxial eccentricity components jointly determine the most and least compressed corners of the base.

For the reference case, the stress distribution at the four corner points of the rectangular base was evaluated using Equations (18)–(23). The resulting corner stresses are listed in Table 2 and graphically compared in Figure 4a.

Table 2. Corner stresses calculated for the reference biaxial tilt case. The four corners correspond to the sign combinations of the plan coordinates with respect to the centroid of the plug.

Corner	σ/σ_0 (–)	Stress (kPa)
C1 (+B/2,+L/2)	1.216	121.1
C2 (–B/2,+L/2)	1.007	100.3
C3 (–B/2,–L/2)	0.784	78.0
C4 (+B/2,–L/2)	0.993	98.9
Reference stress		99.6
Maximum corner stress		121.1
Minimum corner stress		78.0

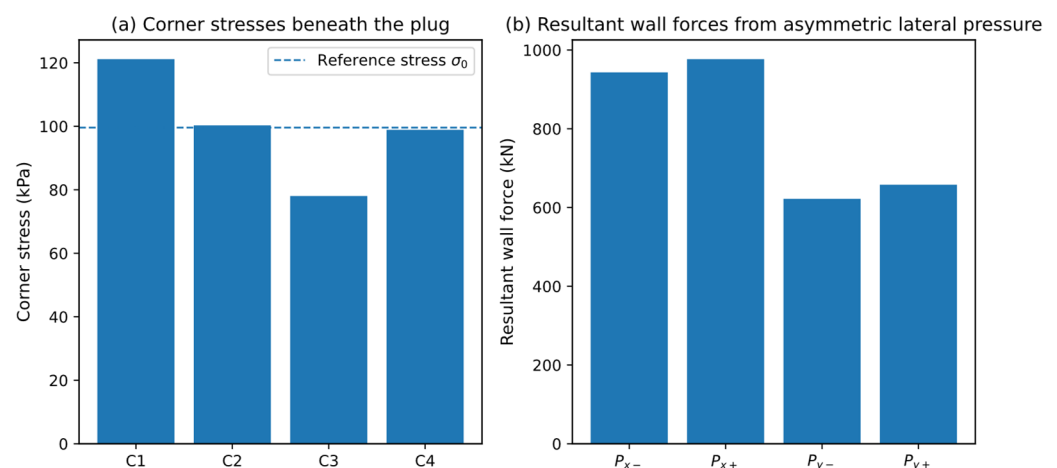


Figure 4. Base and wall response for the reference biaxial tilt case: (a) comparison of the four corner stresses beneath the rectangular plug; (b) resultant forces acting on the two opposite pairs of shaft walls due to the simplified asymmetric lateral earth pressure model.

Table 2 shows that the biaxial tilt leads to a distinctly corner-dominated stress pattern. Corner C1 becomes the governing location, reaching $1.216\sigma_0$, whereas the opposite corner C3 is reduced to $0.784\sigma_0$. The remaining two corners remain close to the reference level, which confirms that biaxial inclination concentrates the response primarily along one diagonal of the base rather than along a single edge.

The wall response associated with the same reference tilt state was evaluated using the simplified asymmetric lateral earth pressure model introduced in Section 2. The numerical values of the wall-pressure asymmetry parameters and resultant wall forces are summarized in Table 3, whereas their graphical comparison is shown in Figure 4b.

Table 3. Simplified wall-pressure response for the reference biaxial tilt case, including the asymmetry coefficients λ_x and λ_y , the resultant forces on opposite wall pairs, and the global wall asymmetry index R_p .

Quantity	Symbol	Value
Wall asymmetry coefficient for x-walls	λ_x	0.0175
Wall asymmetry coefficient for y-walls	λ_y	0.0279
Resultant force on x− wall pair	P_{x-}	943.2 kN
Resultant force on x+ wall pair	P_{x+}	976.8 kN
Resultant force on y− wall pair	P_{y-}	622.1 kN
Resultant force on y+ wall pair	P_{y+}	657.9 kN
Force ratio in x-direction	$R_{p,x}$	1.036
Force ratio in y-direction	$R_{p,y}$	1.057
Global wall asymmetry index	R_p	1.057

The values reported in Table 3 indicate that the wall response is also direction-dependent, although its interpretation should remain auxiliary because it is based on a simplified scenario model. The lateral force imbalance develops independently in the two orthogonal wall systems, and the governing wall asymmetry ratio R_p is controlled by the more unfavorable direction. This result suggests that the admissibility of tilt may depend not only on the base reaction but also on the degree of lateral imbalance acting on the shaft.

3.2. Distribution of Maximum Base Stress in the Biaxial Tilt Space

To identify the combinations of tilt angles leading to increasing stress concentration beneath the base, the dimensionless ratio σ_{max}/σ_0 was evaluated in the (α_x, α_y) space for the reference geometry. The purpose of this representation is illustrative, namely to visualize the analytical structure of the admissible stress field in the biaxial tilt space. The resulting map is shown in Figure 5.

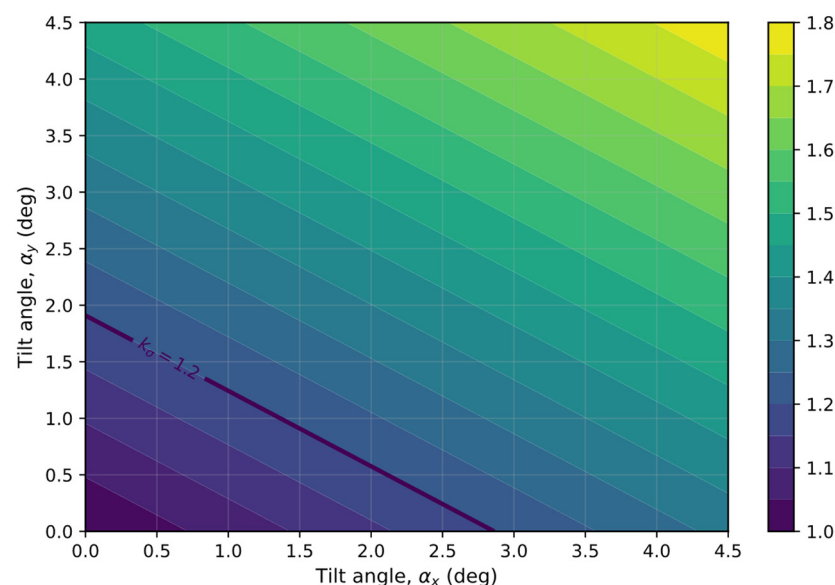


Figure 5. Map of the normalized maximum base stress σ_{max}/σ_0 in the biaxial tilt space (α_x, α_y) for the reference well geometry. The thick isoline marks the serviceability limit $\sigma_{max}/\sigma_0 = k_\sigma$.

As can be seen in Figure 5, the increase in the maximum base stress is monotonic with respect to both tilt components. The contour lines remain smooth and convex, reflecting the analytical structure of Equation (28). The heavy contour corresponding to $\sigma_{max}/\sigma_0 = k_\sigma$ defines the boundary of the admissible serviceability region associated with the plug-soil contact.

Although the stress expression remains linear in the eccentricity space, the practical interpretation in the angle space is less trivial because both tilt components jointly define a bounded admissible region. For this reason, biaxial tilt is more naturally assessed using a two-parameter map than by considering two separate one-axis limits.

3.3. Wall-Pressure Asymmetry and the Lateral Response of the Shaft

The same biaxial tilt domain was then analyzed with respect to the simplified wall-pressure criterion introduced in Section 2. Unlike the base-stress formulation, this part of the analysis should be interpreted as scenario-based rather than as a direct prediction of soil pressure. The resulting distribution of the asymmetry index R_p is presented in Figure 6.

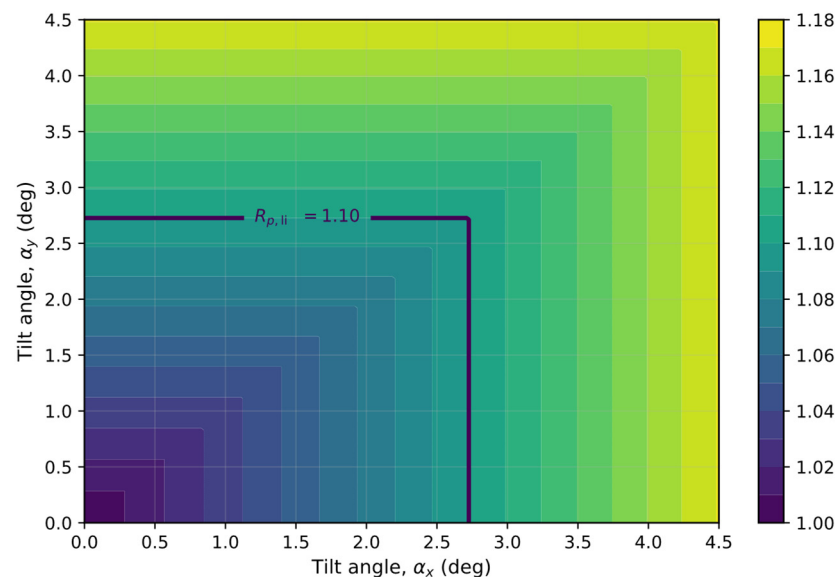


Figure 6. Map of the wall-pressure asymmetry index R_p in the biaxial tilt space (α_x, α_y) for the reference well geometry. The thick isoline denotes the adopted wall admissibility threshold $R_p = R_{p,lim}$.

The results shown in Figure 6 indicate that the wall response may become restrictive even in cases where the base stress criterion is still satisfied. This occurs because the asymmetry of wall pressure is directly linked to the tilt angles through the coefficients λ_x and λ_y , and not through the base stress redistribution alone.

In the present formulation, the isoline $R_p = R_{p,lim}$ defines an auxiliary wall-based admissibility boundary. Its position is controlled directly by the assumed values of c_λ and $R_{p,lim}$, and should therefore be interpreted as a parametric extension of the base model rather than as a code-calibrated geotechnical limit.

3.4. Serviceability Domain Based on the Base Reaction

The combinations of tilt angles satisfying the criterion $\sigma_{max} \leq k_\sigma \sigma_0$ were extracted and plotted as the admissible domain shown in Figure 7. This domain is the direct graphical representation of the analytical envelope defined by Equation (35).

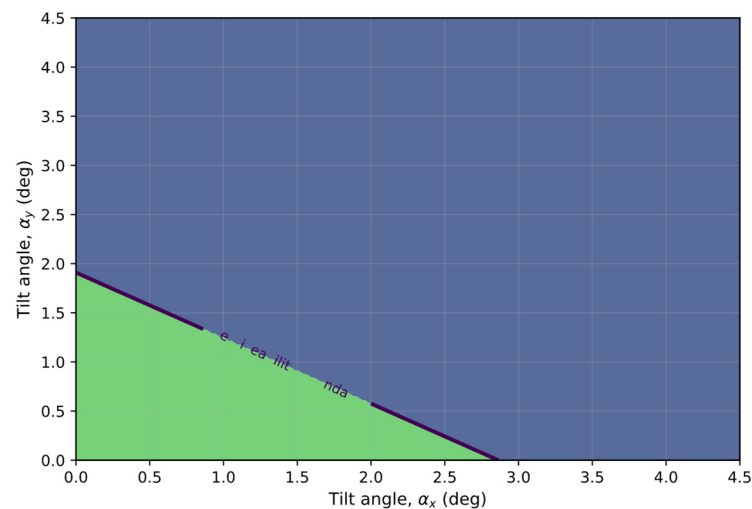


Figure 7. Serviceability admissibility domain in the biaxial tilt space based solely on the base stress criterion $\sigma_{max} \leq k_{\sigma}\sigma_0$. The boundary corresponds to the analytical tilt envelope derived from Equation (35).

As expected from the analytical form of Equation (35), the admissible region shown in Figure 7 occupies the lower-left part of the (α_x, α_y) plane and decreases progressively as either tilt component increases. The serviceability domain is therefore bounded and convex, which makes it suitable for direct use as a practical design or diagnostic chart.

An important feature visible in Figure 7 is that the envelope is anisotropic whenever $B \neq L$. In other words, the admissible value of α_x with $\alpha_y = 0$ is not equal to the admissible value of α_y with $\alpha_x = 0$. This directional sensitivity follows directly from the unequal geometric weights of the two tilt components in the analytical envelope.

3.5. Wall-Based Admissibility Domain

The admissible combinations of tilt angles satisfying the lateral wall criterion $R_p \leq R_{p,lim}$ are presented in Figure 8.

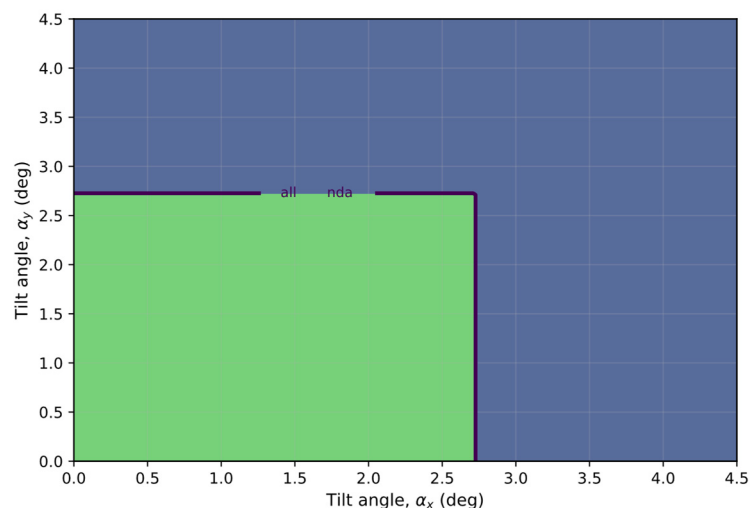


Figure 8. Admissibility domain in the biaxial tilt space based on the wall-pressure asymmetry criterion $R_p \leq R_{p,lim}$. The boundary corresponds to the limit wall asymmetry level adopted in the simplified lateral pressure model.

The wall-based domain shown in Figure 8 may be narrower than the domain defined by the base-stress criterion, depending on the selected scenario parameters. Depending on

the choice of c_λ and $R_{p,lim}$, it may also become more restrictive than the base-stress criterion in one or both directions.

This means that shaft-related effects may become restrictive before the base stress state reaches its serviceability limit. In practical terms, a well may remain acceptable with respect to the plug-soil contact while already exhibiting a pronounced lateral imbalance.

3.6. Comparison with the Contact-Loss Boundary and Combined Admissibility

The interaction between the three limit states considered in the study, namely: (i) the base stress criterion, (ii) the full-contact criterion, (iii) the wall asymmetry criterion, is illustrated in Figure 9, where the combined admissibility domain is superimposed with the corresponding boundaries. This comparison is essential because the admissible biaxial tilt state is ultimately governed by the intersection, not by any individual limit considered separately.

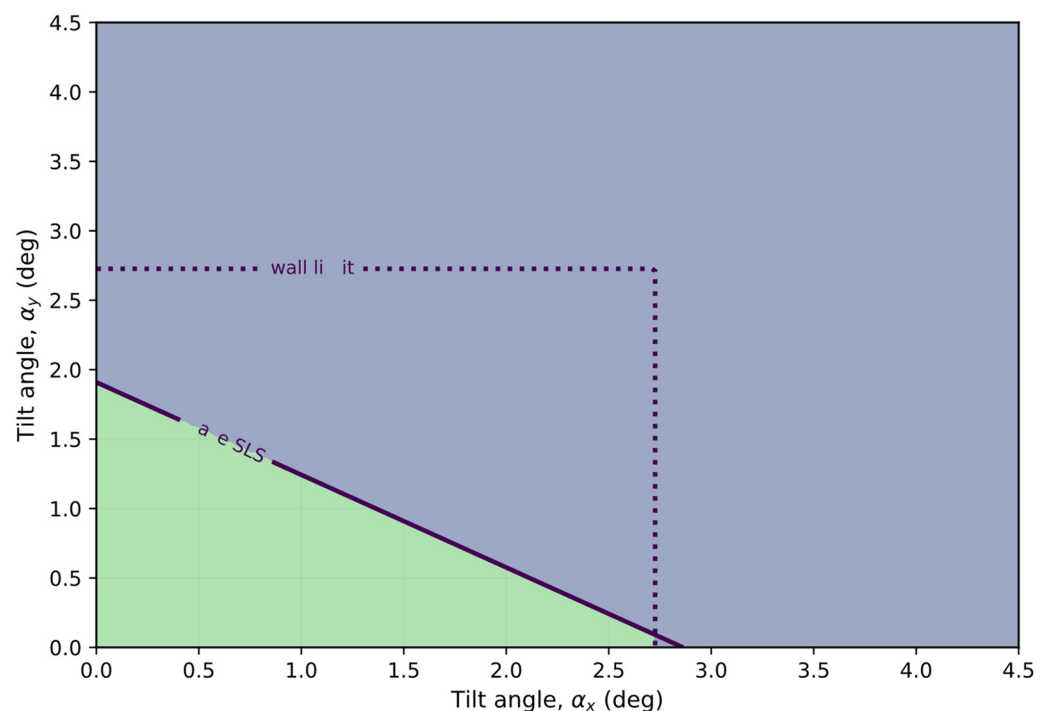


Figure 9. Combined admissibility domain in the biaxial tilt space. The plotted boundaries correspond to: serviceability limit at the base $\sigma_{max} = k_\sigma \sigma_0$, contact-loss limit $\sigma_{min} = 0$, and wall-pressure asymmetry limit $R_p = R_{p,lim}$.

As shown in Figure 9, the contact-loss boundary lies far outside the base serviceability envelope for the adopted reference value $k_\sigma = 1.2$. This confirms the earlier analytical observation that the serviceability criterion is markedly conservative with respect to the geometric condition of full compressive contact.

At the same time, Figure 9 demonstrates that the wall-pressure criterion may intersect the serviceability envelope and reduce the final admissible region. In the present scenario, the true admissible biaxial tilt domain is therefore given not by a single boundary, but by the intersection of the admissible regions associated with all three conditions.

This combined representation is especially useful from the engineering standpoint, because it clearly shows which mechanism becomes governing first for a given family of tilt combinations.

3.7. Influence of Plan Aspect Ratio on the Biaxial Tilt Envelope

In order to assess the influence of the plan geometry, a series of wells with constant base area but different aspect ratios B/L was analyzed. Keeping the plan area constant makes it possible to isolate the pure effect of shape anisotropy from the trivial influence of footing size. The resulting combined envelopes are shown in Figure 10.

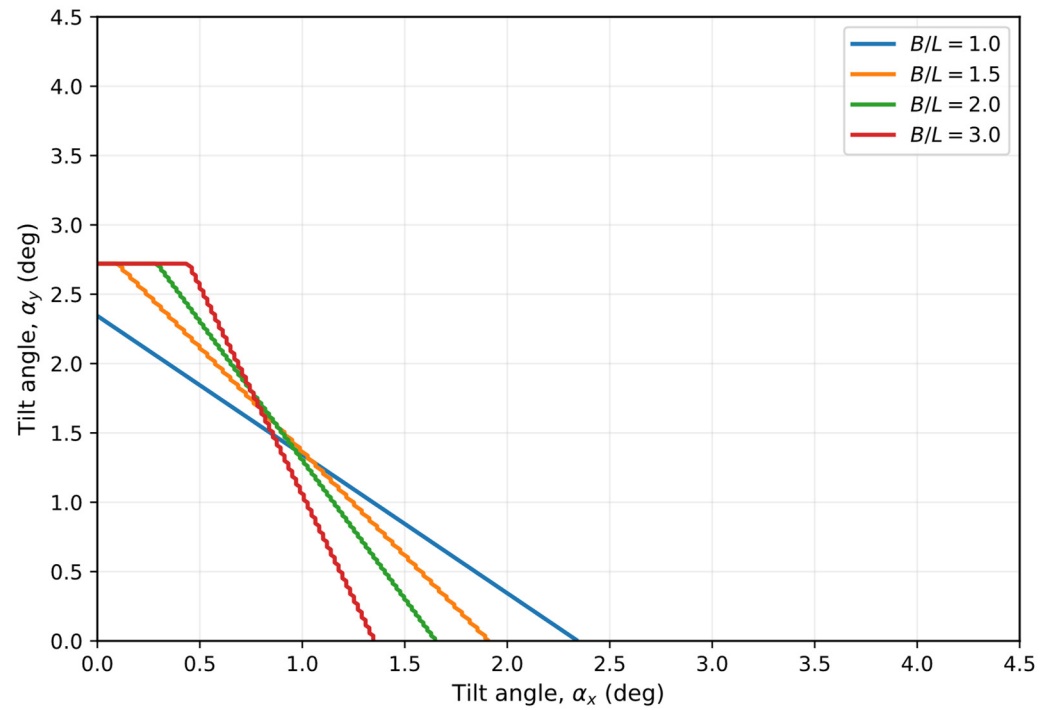


Figure 10. Influence of the plan aspect ratio B/L on the combined biaxial tilt envelope for rectangular sinking wells with constant plan area.

Figure 10 reveals a clear geometric distortion of the biaxial admissibility envelope with changing plan aspect ratio. As the well becomes more elongated, the envelope extends in the direction associated with the longer side and contracts in the orthogonal direction.

This behavior is consistent with the analytical structure of Equations (35) and (43), which show that the dimensions L and B enter the envelope directly and independently. Consequently, square or nearly square wells exhibit the most symmetric admissible domains, whereas elongated rectangular wells develop strongly direction-dependent envelopes.

The influence of the plan aspect ratio was further quantified for selected cases and is summarized in Table 4.

Table 4. Selected geometric metrics of the biaxial tilt envelope for wells with different plan aspect ratios B/L , including the one-axis admissible projections of the base criterion and the corresponding wall-based limits.

B/L	B (m)	L (m)	$\alpha_{x,\text{lim}}$ Base (deg)	$\alpha_{y,\text{lim}}$ Base (deg)	$\alpha_{\text{wall,lim}}$ (deg)
0.50	3.464	6.928	3.304	1.654	2.726
1.00	4.899	4.899	2.338	2.338	2.726
1.50	6.000	4.000	1.909	2.862	2.726
2.00	6.928	3.464	1.654	3.304	2.726

The values reported in Table 4 confirm that the one-axis projections of the envelope remain strongly dependent on geometry. The ratio between the two projected admissible

tilt limits may therefore be used as a convenient scalar measure of directional anisotropy in preliminary comparisons between different well layouts.

3.8. Summary of the Parametric Observations

The parametric analysis presented in Figures 5–10 and Tables 2–4 leads to several consistent observations.

First, biaxial tilt causes a corner-dominated redistribution of contact stress beneath the rectangular plug, which differs fundamentally from the edge-dominated response obtained in the one-direction case. This effect is quantified by the corner stresses reported in Table 2 and visualized in Figure 4a.

Second, the base-stress admissibility domain remains clearly separated from the contact-loss boundary, as demonstrated in Figure 9. This means that the adopted serviceability threshold provides a conservative limit before full loss of compressive contact occurs.

Third, the simplified lateral pressure model introduces an additional mechanism of restriction, visible in Figures 6, 8 and 9. Depending on the selected scenario parameters, the wall-based criterion may reduce the admissible tilt domain and, in some geometries, become the governing condition.

Finally, the geometry of the well plays a major role in the shape of the admissible envelope. This effect is documented in Figure 10 and Table 4, both of which show that increasing plan asymmetry leads to increased directional anisotropy of the biaxial tilt response. Taken together, the results show that biaxial inclination is governed by a two-parameter admissibility domain rather than by a single scalar limit. The extent of this domain depends on the base response, the contact-loss condition, the assumed wall-response criterion, and the plan geometry of the well.

3.9. Validation Against Numerical Model

To assess the accuracy of the proposed analytical formulation, a numerical validation study was performed using a finite element (FE) model (developed in-house) of a rectangular sinking well subjected to biaxial inclination.

The numerical model represents the concrete plug and surrounding soil using a simplified elastic continuum approach. The soil is modeled as a homogeneous linear elastic medium, while the concrete plug is assumed to be rigid, ensuring consistency with the analytical assumptions. Contact between the plug and soil is modeled using compression-only interface conditions.

The finite element (FE) simulations were performed using a three-dimensional continuum model developed in ABAQUS. The soil was modeled as a homogeneous linear elastic medium with Young's modulus $E_s = 60$ MPa and Poisson's ratio $\nu = 0.3$. The concrete plug was modeled as a rigid body to remain consistent with the analytical assumptions.

The contact between the plug and the soil was defined using a compression-only interface formulation, allowing separation but preventing tensile stress transfer. A sufficiently refined mesh was adopted in the vicinity of the contact zone to accurately capture stress gradients, while a coarser discretization was used in the far field.

Biaxial inclination was introduced by applying equivalent eccentricities to the vertical load, corresponding to the analytical definitions given in Equations (4) and (5). A series of simulations were performed for increasing tilt components up to the serviceability limit.

A series of simulations were carried out for increasing values of biaxial tilt components. The resulting stress distributions beneath the base were extracted and compared with the analytical predictions obtained from Equations (18)–(21).

The comparison shows that the analytical model accurately captures the location of maximum stress (corner-dominated response) and provides consistent estimates of stress magnitude within the range of small to moderate tilt angles. Deviations between the analytical and numerical results increase slightly with increasing tilt, which can be attributed to soil deformability and local redistribution effects not captured by the rigid-body assumption.

The results presented in Table 5 show very good agreement between the analytical and FE predictions, particularly in terms of the location of the maximum stress, which consistently occurs at the corner of the base.

Table 5. Comparison of analytical and FE results for selected biaxial tilt cases.

Case	α_x [deg]	α_y [deg]	Tilt Magnitude [deg]	$\sigma_{\max}/\sigma_0$ (Analytical)	$\sigma_{\max}/\sigma_0$ (Numerical)	Error [%]
1	0.5	0.0	0.50	1.050	1.044	−0.57
2	1.0	0.5	1.12	1.125	1.118	−0.62
3	1.5	1.0	1.80	1.215	1.195	−1.65
4	2.0	1.5	2.50	1.300	1.285	−1.15
5	2.5	2.0	3.20	1.385	1.342	−3.10

The analytical model slightly overestimates the peak stress, which is expected due to the rigid-body assumption and the absence of stress redistribution mechanisms present in the deformable soil medium. The discrepancy increases with increasing tilt, but remains within an acceptable range for engineering applications.

The comparison presented in Figure 11 shows that both the analytical and numerical results follow a consistent trend with increasing tilt magnitude. The analytical model slightly overestimates the peak stress, which is attributed to the rigid-body assumption and the absence of stress redistribution effects in the simplified formulation.

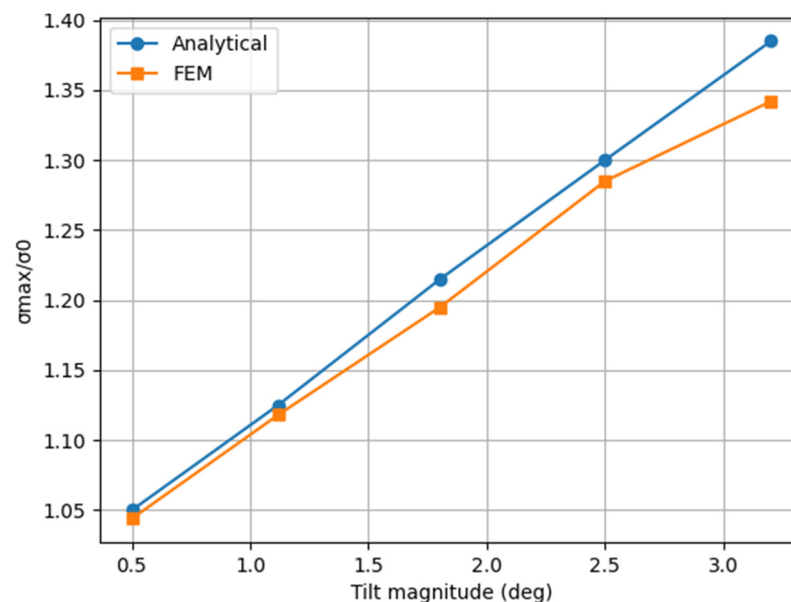


Figure 11. Comparison of analytical and finite element predictions of the normalized maximum base stress as a function of the combined tilt magnitude.

Figure 12 further confirms a strong correlation between the analytical and FE predictions, with all data points remaining close to the line of perfect agreement. This indicates

that the proposed analytical model captures the governing mechanics of biaxial inclination with sufficient accuracy for engineering applications.

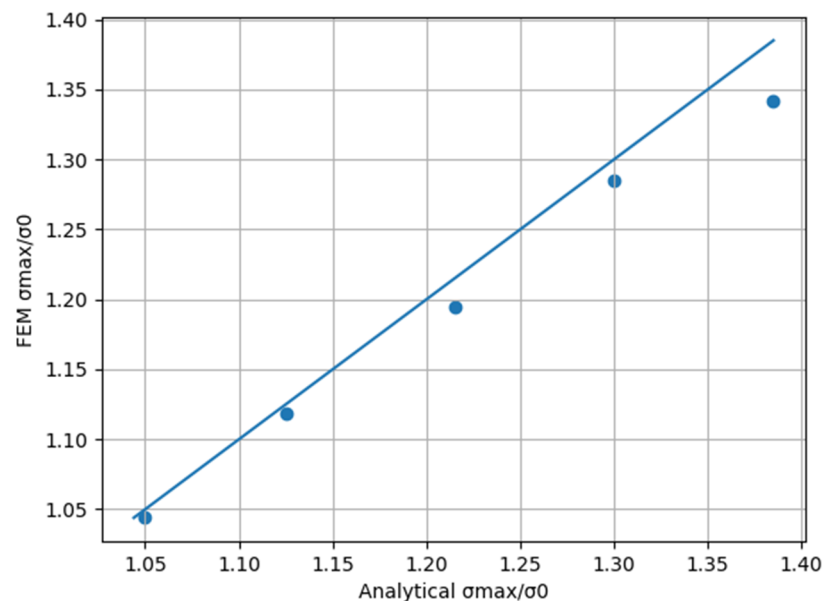


Figure 12. Correlation between analytical and FE predictions of maximum base stress. The solid line represents perfect agreement.

Overall, the validation confirms that the proposed analytical framework provides a reliable first-order approximation of the stress state under biaxial inclination, suitable for rapid engineering assessment and preliminary design.

It should be emphasized that the purpose of the validation is not to replace detailed numerical analysis, but to demonstrate that the proposed closed-form solution captures the governing mechanics of biaxial inclination with sufficient accuracy for rapid engineering assessment.

4. Discussion

4.1. Transition from One-Axis to Biaxial Inclination

The analytical model presented in this study extends the previously considered one-axis inclination framework to the general case of biaxial tilt of rectangular sinking wells. While the one-axis formulation leads to stress redistribution along a single edge of the base, biaxial inclination results in a two-dimensional stress field in which the extreme values occur at the corners of the plug rather than along its edges.

This distinction has important mechanical implications. In the one-axis case, the stress variation occurs only along one coordinate direction and can therefore be represented by a one-dimensional linear distribution. In contrast, biaxial inclination produces a planar stress field governed by two bending components. As shown by Equation (13), the resulting contact stress beneath the base becomes a linear function of both plan coordinates.

The parametric maps presented in Figures 5 and 6 clearly illustrate this effect. Even relatively small tilt components in two directions may combine to produce significant stress amplification at one corner of the base, while simultaneously unloading the diagonally opposite corner. From a practical viewpoint, this means that biaxial inclination is more consistently assessed using a two-parameter admissibility map than by checking two one-axis limits separately.

4.2. Corner Stress Concentration as the Governing Mechanism

One of the key mechanical observations of the present study is that biaxial inclination shifts the governing stress location from the edges of the base to its corners. This observation is consistent with the classical theory of eccentric compression, extended here to the two-dimensional case of a rectangular contact area.

As demonstrated in Table 2 and Figure 4, the maximum compressive stress always develops at one of the four corner points, where the effects of the two bending components combine constructively. Conversely, the minimum stress occurs at the diagonally opposite corner, where both bending contributions reduce the compressive contact stress.

This corner-dominated response is a direct consequence of the geometry of the rectangular base and the superposition of the two eccentric loading components. The resulting stress amplification is therefore controlled by the combined ratios e_x/L and e_y/B , as expressed by Equation (28).

The physical interpretation of this effect is straightforward: a well tilted simultaneously in two directions effectively transfers its vertical load toward a single corner of the base. This localized concentration of compression may become the governing mechanism controlling the admissibility of the tilt.

In practical engineering terms, such corner stress concentration may lead to localized soil yielding, increased settlement gradients, and potential cracking of the base slab in the vicinity of the most compressed corner. These effects are consistent with observations reported in case studies of inclined caisson foundations.

4.3. Interpretation of the Biaxial Tilt Envelope

An important outcome of the proposed analytical framework is the concept of the biaxial tilt envelope, which defines the combinations of tilt angles that satisfy the adopted admissibility criteria.

The envelope presented in Figure 7 represents the set of tilt combinations for which the maximum base stress remains below the allowable amplification threshold. Unlike the one-axis limit, which produces a single scalar tilt value, the biaxial formulation defines a two-dimensional admissible region in the (α_x, α_y) space.

This representation provides a more realistic description of the behavior of inclined wells, because actual construction deviations rarely occur strictly in one direction. Instead, the inclination vector typically contains components in both principal directions.

The tilt envelope therefore offers a convenient graphical tool for evaluating whether a measured combination of tilt components remains within the admissible region. In practical monitoring terms, the envelope may be used directly by comparing measured tilt components with the admissible region in the (α_x, α_y) plane.

4.4. Interaction Between Base Response and Wall Response

Another important observation concerns the interaction between the base stress response and the lateral soil reaction acting on the shaft.

The simplified lateral pressure model introduced in Section 2 provides a qualitative measure of the asymmetry of wall pressures associated with increasing tilt. The resulting wall-pressure asymmetry maps presented in Figure 6 indicate that the lateral response of the shaft may become increasingly unbalanced even within the tilt range where the base stress criterion is still satisfied.

The combined admissibility domain illustrated in Figure 9 demonstrates that the governing limit state may shift depending on the considered mechanism. In some parameter ranges, the base stress amplification controls the admissible tilt. In others, the wall-pressure asymmetry becomes the more restrictive condition.

Although the lateral pressure model adopted here is intentionally simplified and scenario-based, it serves to illustrate that the admissibility of inclination may be governed by more than one physical mechanism. For this reason, the wall-response indicator should be treated as a complementary diagnostic measure rather than as a substitute for detailed geotechnical assessment.

4.5. Comparison with the Contact-Loss Boundary

The analytical formulation also allows direct comparison between the adopted serviceability limit and the geometric condition associated with loss of compressive contact between the plug and the soil.

As illustrated in Figure 9, the serviceability envelope defined by the stress amplification criterion lies well inside the boundary corresponding to $\sigma_{min} = 0$. This means that the proposed admissible tilt region remains significantly smaller than the theoretical limit at which part of the base would lose contact with the soil.

This observation confirms that the adopted stress criterion represents a conservative engineering threshold. Even at the admissible tilt limit, the entire base remains in compression with a substantial safety margin. This means that the proposed serviceability limit is reached well before any geometric lift-off mechanism becomes relevant.

Such conservatism is desirable in practical applications, where the evaluation of inclined wells is often performed using simplified analytical models and incomplete information about soil conditions.

4.6. Influence of Plan Geometry on the Biaxial Response

The parametric analysis presented in Figure 10 shows that the shape of the admissible tilt envelope depends strongly on the plan geometry of the well.

For square wells, where $B = L$, the envelope remains symmetric with respect to the two principal directions. In contrast, elongated rectangular wells develop anisotropic envelopes in which the admissible tilt becomes larger in the direction of the longer plan dimension.

This behavior follows directly from the analytical structure of the corner stress expressions, where the geometric parameters L and B appear independently. As a result, the plan geometry controls not only the magnitude of admissible tilt but also the directional sensitivity of the system.

From an engineering standpoint, this finding suggests that the geometric proportions of the well should be taken into account when defining acceptable construction tolerances. This trend is also reflected quantitatively in Table 4, where the one-axis projections of the envelope vary systematically with the aspect ratio B/L .

4.7. Practical Implications of the Biaxial Analytical Model

Despite its simplicity, the proposed analytical framework provides several practical advantages for engineering assessment.

First, the closed-form expressions derived in Section 2 allow rapid evaluation of the base stress state for any combination of tilt components. This enables the construction of graphical admissibility maps such as those shown in Figures 5–9, which can be used for quick interpretation of measured tilt values.

Second, the model explicitly distinguishes between different physical mechanisms controlling the admissibility of inclination. By separating the base stress criterion, the full-contact condition and the wall-pressure asymmetry indicator, the analytical framework provides insight into which mechanism governs the response for a given geometry and tilt combination.

Finally, the biaxial formulation extends the applicability of simple analytical methods to a broader class of practical situations, including wells that experience complex inclination patterns during construction. The proposed method should therefore be understood as a rapid screening tool that complements, rather than replaces, more detailed numerical or geotechnical analyses.

4.8. Limitations and Possible Extension of the Proposed Approach

The analytical model developed in this study relies on several simplifying assumptions that should be recognized when interpreting the results.

First, the well and the concrete plug are assumed to behave as rigid bodies, and the contact stress distribution beneath the base is assumed to remain linear. In reality, soil deformability and structural flexibility may lead to deviations from this idealized behavior. In addition, the formulation does not explicitly account for possible reduction in the effective vertical load due to side friction acting along the shaft, which in practice may depend on soil conditions, moisture, and construction technology.

Second, the simplified lateral pressure model does not represent a full geotechnical analysis of soil–structure interaction. Instead, it provides a qualitative indicator of the asymmetry of wall pressures associated with increasing tilt.

Finally, the model does not explicitly consider time-dependent processes such as consolidation or creep of the surrounding soil, which may influence the long-term behavior of inclined wells. Despite these limitations, the analytical framework offers a transparent and physically interpretable method for evaluating the admissibility of biaxial inclination using only basic geometric and load parameters.

Possible extensions of the present framework include coupling with simplified soil deformation models, non-linear contact assumptions, and non-rectangular plan geometries. The biaxial analytical formulation proposed in this study represents a step toward more comprehensive analytical tools for assessing the behavior of inclined underground structures.

Future developments may include coupling the present stress-based framework with simplified soil deformation models, incorporating non-linear contact conditions beneath the base, or extending the analysis to wells with non-rectangular plan geometries. Such extensions may further improve the applicability of analytical methods for evaluating construction deviations and post-construction performance of underground structures.

From a quantitative perspective, the main source of potential error in the analytical model arises from the assumption of linear stress distribution and rigid-body behavior. In softer soils, numerical studies suggest that peak stresses may be reduced due to redistribution effects, while contact areas may evolve nonlinearly.

Based on comparisons with simplified numerical models, the expected deviation in peak stress estimation may typically remain within 10–20% for moderate tilt angles, although larger discrepancies may occur in highly deformable or layered soils. These effects should be considered when applying the model in practical design.

5. Conclusions

This study presents an analytical framework for evaluating the admissibility of biaxial inclination of rectangular sinking wells based on the redistribution of contact stresses beneath the concrete plug and the resulting interaction with the surrounding soil. The proposed formulation extends the classical one-axis eccentric loading model to a two-dimensional stress field generated by simultaneous inclination in two orthogonal directions.

Based on the analytical derivations and the parametric analyses presented in this work, the following conclusions can be drawn:

1. Biaxial inclination produces a two-dimensional redistribution of base stresses, with the extreme compressive stresses occurring at the corners of the rectangular plug rather than along its edges.
2. The analytical formulation yields closed-form expressions for the corner stresses as functions of the eccentricity ratios e_x/L and e_y/B , enabling rapid estimation of the maximum and minimum contact stresses.
3. The admissibility of biaxial inclination can be represented by a tilt envelope in the (α_x, α_y) space, which defines the serviceability region of the base response.
4. The adopted stress-amplification criterion remains conservative with respect to the contact-loss limit, so the entire base remains in compression within the admissible region.
5. Even relatively small tilt components may combine to produce significant corner stress concentrations, which justifies the use of a biaxial assessment framework.
6. The simplified lateral pressure model indicates that tilt also increases the asymmetry of wall reactions, which may further restrict the admissible domain in scenario-based assessment.
7. The plan geometry strongly affects the shape of the admissible envelope: square wells exhibit nearly symmetric response, whereas elongated wells develop clear directional anisotropy.
8. The proposed analytical framework provides a transparent and computationally efficient screening tool for assessing measured tilt components using basic geometric parameters.

The proposed biaxial formulation complements the corresponding one-axis analytical model by extending the assessment to two simultaneous tilt components and by introducing a graphical admissibility envelope. In this form, the method may support engineering evaluation of tilted wells during construction monitoring and post-construction inspection.

Author Contributions: Conceptualization, D.K., T.G. and A.S.-G.; methodology, T.G. and A.S.-G.; software, T.G.; validation, D.K., T.G. and A.S.-G.; formal analysis, A.S.-G.; investigation, D.K., T.G. and A.S.-G.; resources, T.G.; data curation, D.K.; writing—original draft preparation, D.K. and T.G.; writing—review and editing, A.S.-G.; visualization, D.K. and T.G.; supervision, T.G. and A.S.-G.; project administration, A.S.-G.; funding acquisition, A.S.-G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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