# Mechanical calibration of damage enhanced anisotropic constitutive models for free-foils *

**Tomasz Garbowski and Tomasz Łodygowski**
*Institute of Structural Engineering, Poznan University of Technology*
*ul. Piotrowo 5, 60-695 Poznan, Poland*
*e-mail: tomasz.garbowski@put.poznan.pl, tomasz.lodygowski@put.poznan.p*

Abstract

In the present paper the short review of popular anisotropic constitutive models enhanced by the continuum damage models is presented. The attention is restricted to thin anisotropic foils, therefore models in plane stress condition only are examined. The anisotropic models themselves contain rather big number of parameters to be calibrated, therefore the mechanical characterization becomes even more difficult when damage is considered. In the paper the numerical implementations of enhanced anisotropic models into the commercial FE software is presented followed by the description of novel inverse procedure for parameter identification based on biaxial tests, experiment simulations and Digital Image Correlation (DIC).

*Keywords: computational mechanics, orthotropy, continuum damage, digital image correlation, inverse analysis, biaxial tests*

## 1. Introduction

The free-foils, despite they diverse internal structures and thicknesses, are more and more frequently employed in various industries. Fiber metal laminates consist of aluminum layers bonded by fiber reinforced pre-preg layer employed in an aircraft design; layered structures composed of different sheets of materials used in the food packaging industries; carbon fiber-reinforced composites exploited in passive protection of structural elements under static and dynamic loads, to list just a few.

Usually such composites and layered materials have different mechanical behavior when loaded in different directions. This behavior is regarded as anisotropy or, more precise, as orthotropy and can be modeled by various elastic-plastic constitutive models. Here, the popular Hill model [1] and its generalized version (Hoffman model) [2] are investigated, followed by Tsai-Wu model [3].

The work hardening rule employed in the above listed models is first chosen to be isotropic (just one hardening internal variable controls the hardening). Later, the anisotropic hardening is considered [4]: each hardening parameter describes the behavior of its corresponding equivalent stress component independently. In order to capture softening and fracture processes in thin free-foils the local (regularized) and non-local (integral type) isotropic [5] (and orthotropic) damage is additionally investigated.

Mechanical calibration of such complicated models is not an easy task and often requires sophisticated experimental techniques to be employed. Therefore the experimental technique which combines biaxial testing and Digital Image Correlation (DIC) for displacement measurements is presented here for material parameter identification.

## 2. Anisotropic elasto-plasticity

The elastic behavior of anisotropic materials is often described by the linear elasticity law defined by:

$$\dot{\sigma}_{ij} = C^e_{ijkl}\dot{\varepsilon}^e_{kl} \tag{1}$$

where $C^e_{ijkl}$ is the fourth-order elasticity tensor, which accounts for anisotropic effects ranging from transversal isotropy and orthotropy to general anisotropy with 21 elastic constants. The basic assumption in elasto-plasticity using small deformation theory is that the total Cauchy strain-rate tensor, $\dot{\varepsilon}_{ij}$, can be decomposed into a sum of an elastic and a plastic part, i.e.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^p_{ij} \tag{2}$$

in which the plastic strain-rate tensor is determined from the associated plastic flow rule or the normality law defined by:

$$\dot{\varepsilon}^p_{ij} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}} \tag{3}$$

where $\sigma_{ij}$ is the Cauchy stress tensor, $\dot{\lambda}$ is a plastic multiplier and $f$ is the yield function which can be defined under the isotropic work-hardening assumption by:

$$f = \sigma_{eq}\left(\sigma_{ij}\right) - \bar{\sigma}(\varepsilon^p_{eq}) \leq 0 \tag{4}$$

where $\sigma_{eq}$ denotes a scalar measure of the effective stress, $\bar{\sigma}$ is a scalar hardening function and $\varepsilon^p_{eq}$ is an equivalent plastic strain, which in rate form reads: $\dot{\varepsilon}^p_{eq} = (\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij})^{1/2}$. The effective stress expresses the severity of the present stress state, while the hardening function monitors the present location of the yield surface. The hardening function may consist of a single constant (elastic perfectly plastic materials) or a function of internal variables (hardening materials). The criteria for plastic deformation to occur in hardening materials is that $f = 0$ and $\dot{\lambda} > 0$ are satisfied simultaneously.

### 2.1. *The Hill model*

The Hill criterion has been introduced as an orthotropic extension of the standard Huber-Mises-Henky (HMH) criterion in order to model the anisotropy often found in formed steel. With $\sigma_{ij}$ denoting the stress tensor components on an orthonormal basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ whose vectors coincide with the principal axes of plastic orthotropy. The yield function associated with the Hill criterion is shown in the Table 1, where $\bar{\sigma}$ is the relative yield stress (a non-dimensional scalar) which defines the size (state of hardening) of the yield surface in the stress space. The constants $F_i$ are the functions of $\sigma^0_{ij}$ which are the generally distinct (but equal in tension and compression) normal and shear yield stresses. The initial state of the material is assumed when $\bar{\sigma}(\varepsilon^p_{eq} = 0) = 1$.

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Table 1: Anisotropic yield functions

| Name | Yield function |
|---|---|
| Hill | $f(\boldsymbol{\sigma}, \varepsilon^p_{eq}) = F_1 (\sigma_{11} - \sigma_{22})^2 + F_2\sigma^2_{22} + F_3\sigma^2_{11} + F_4\sigma^2_{12} - [\bar{\sigma}(\varepsilon^p_{eq})]^2$ |
| Hoffman | $f(\boldsymbol{\sigma}, \varepsilon^p_{eq}) = C_1 (\sigma_{11} - \sigma_{22})^2 + C_2\sigma^2_{22} + C_3\sigma^2_{11} + C_4\sigma^2_{12} + C_5\sigma_{11} + C_6\sigma_{22} - [\bar{\sigma}(\varepsilon^p_{eq})]^2$ |
| Tsai-Wu | $f(\boldsymbol{\sigma}, \varepsilon^p_{eq}) = F_1\sigma_{11} + F_2\sigma_{22} + F_{11}\sigma^2_{11} + F_{22}\sigma^2_{22} + F_{44}\sigma^2_{12} + 2F_{12}\sigma_1\sigma_2 - [\bar{\sigma}(\varepsilon^p_{eq})]^2$ |

Since general straining of an initially orthotropic material is usually expected to change the yield stresses in different directions by different amounts, or even lead to loss of orthotropy, the model resulting from the above assumptions provides only a first approximation to the phenomenon of hardening.

### 2.2. *Generalized Hill model (Hoffman model)*

For many materials (e.g. composite, worked metals, paper and paperboards) a marked difference is observed between yield stress levels in tension and compression (the Bauschinger effect). In order to model such effects in orthotropic materials, Hoffman proposed an extension to the Hill criterion described by the yield function depicted in the Tab. 1, where $C_i$, analogous to that of the Hill criterion) are defined as functions of $\sigma^{t0}_{ij}$ and $\sigma^{c0}_{ij}$, with t – tension, c – compression.

The effect of (isotropic) hardening can also be incorporated into the Hoffman criterion by assuming $\bar{\sigma}(\varepsilon^p_{eq})$ to be a function of the accumulated plastic strain.

### 2.3. *Anisotropic hardening*

In the case of anisotropic hardening of the above models, the individual yield values $\sigma^0_{ij}$ do not remain constant but depend on a corresponding hardening parameter $\sigma^y_{ij}(\kappa_{ij})$. To avoid a coupling between an isotropic type of hardening and an anisotropic type the equivalent stress $\bar{\sigma}$ in Eq. (4) and in Tab. 1 remains constant. Here it is assumed that the equivalent stress $\bar{\sigma}$ is equal to the virgin yield strength of the material in the first material direction: $\bar{\sigma} = \sigma^0_{11}(\kappa_{11} = 0)$.

## 3. Damage

In order to describe the loss of material integrity due to propagation and coalescence of micro-cracks which leads to degradation of material stiffness the continuum damage is employed here, where the relation between the stress and elastic strain reads:

$$\sigma_{ij} = (1 - \omega) C^e_{ijkl}\varepsilon^e_{kl} \tag{5}$$

with $\omega$ being a damage parameter (scalar in isotropic damage) ranging from 0 (virgin material) to 1 (completely damage material). The two mesh-independent implementations are presented herein as follow: (i) model with a fracture energy regularization technique and (ii) non-local integral type of damage model [5], by weighted averaging of the certain variable over the spatial neighborhood of each point of interest. In the nonlocal damage model adopted here, non-locality enters the constitutive equations through the definition of non-local scalar measure of equivalent plastic strain:

$$\tilde{\varepsilon}^p_{eq}(\mathbf{x}) = \frac{\int_\Omega \psi(\mathbf{y}, \mathbf{x})\, \varepsilon^p_{eq}(\mathbf{y}) d\Omega(\mathbf{y})}{\int_\Omega \psi(\mathbf{y}, \mathbf{x})\, d\Omega(\mathbf{y})} \tag{6}$$

where the $\psi$ is chosen here to be homogeneous and isotropic Gaussian weight function:

$$\psi(\rho) = \frac{1}{2\pi l^2} \exp\left(-\frac{\rho^2}{2l^2}\right) \tag{7}$$

with $\rho$ being the distance between the points $\mathbf{y}$ and $\mathbf{x}$ and $l$ the material parameter called "length scale".

In such non-local formulation the damage parameter is the function of non-local measure of equivalent plastic strain over certain domain: $\omega = \omega(\tilde{\varepsilon}^p_{eq})$. Further enhancements of the damage model investigated here are as follow: (i) an orthotropic damage propagation (no longer scalar $\omega$ but tensor $\mathbf{w}$ controls damage in the material through nonlocal internal variable $\tilde{\kappa}_{ij}$); (ii) strain-rate dependence of damage evolution, namely:

$$\mathbf{w} = w_{ij}(\tilde{\kappa}_{ij}, \dot{\varepsilon}_{ij}). \tag{8}$$

## 4. Parameter identification

The minimization algorithm employed here for the parameters characterization is a first-order, deterministic, butch (not sequential) Trust Region Algorithm (TRA). The discrepancy between experimental and numerical data is minimized, in least-square sense, by an incremental updates of the sought constants embedded in a finite element (FE) model.

The Hill and the Hoffman model benefit from a relatively simple identification of material parameters (usually a set of uniaxial tests performed in the directions of orthotropy is sufficient for model characterization). In contrast, the Tsai-Wu plasticity model (Tab. 1) suffers from a strong sensitivity of the measured material parameters and the calibration requires additional biaxial test to be utilized.

Identification of the parameters in the anisotropic elasto-plastic models with anisotropic hardening coupled to damage cannot be done by making use of the simple uniaxial tests anymore. In those cases the novel experimental procedure, namely biaxial tests combined with DIC measurements [6] are necessary in order to successfully identify the model parameters. This experimental tools allows to calibrate the above listed models efficiently and robust with just few experiments, although the number of the material constants to be characterized usually exceed twenty.

## References

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