Reliable mechanical characterization of layered pavement structures

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Abstract

In order to obtain the geometrical and mechanical properties of an axisymmetric layered system a numerical high performance model is constructed built on the data provided by the falling-weight-deflectometer (FWD). This apparatus allows for fast non-destructive testing and consists of a falling mass hitting the pavement and a set of sensors collecting the vertical surface oscillations at different distances. The objective of this research is at first to construct a forward model for simulating wave propagation into a layered media. By adopting the spectral element technique only one element per layer is required and computational efficiency is guaranteed, especially when compared to standard dynamic FEM procedures. The main numerical efforts at this stage are the attainment of the dynamic complex stiffness matrix for each discrete frequency and wave number and the solution of the related linear system. Subsequently an inverse method based on the framework of the forward model is proposed for the calculation of the unknown parameters. The procedure is now concerned with the minimization of the objective function which quantifies the difference between computed quantities and their measured counterpart. The solution of the nonlinear system of equations is searched through iterative methods and finally results of different algorithms are compared.

Keywords: falling weight deflectometer, spectral element method, inverse analysis

1. Introduction

Pavement consists of few layers of asphalt, placed on granular sub-base and base, all of them are specified by their thickness and elastic properties. However the initial properties are gradually changing during an extensive overloading of the road structure, which enforces frequent in-situ examinations of the deteriorated pavement characteristics. This is mainly done by using nondestructive tests enhanced by the numerical or analytical models and inverse analysis.

Mechanical identification of layered pavement structures is an actual and an important problem which attracts many researchers around the world. In the literature one can find many approaches based on an inverse procedure combined with a dynamic test (e.g. falling-weight-deflectometer - FWD) and analytical or numerical models. In most cases the static analysis are used, which creates an obvious divergence between a purely dynamic response of the real structure and its numerical quasi-static model. This problem was partially solved by introducing the filtered (i.e., zero-frequency) force and deflection values [2] into the inverse procedure. Other solutions usually base on dynamic FE models, which unfortunately are very costly and therefore impractical in real life applications.

2. Problem formulation

The proposed here procedure use a spectral element method (SEM) instead of dynamic FE models. The main advantage, beside a substantial decrease of the computational time, is a significant increase of an experimental data to be incorporated into the inverse analysis. This is mainly because one can now freely sample the dynamic response not only in space but also in time. Moreover the bigger amount of data creates a great possibility to regularized usually ill-posed inverse procedure.

In the following subsections the forward model based on SEM is briefly described followed by the short introduction to inverse analysis and concluding remarks.

2.1. Wave equations

A vertical impulse load acting on a homogeneous isotropic half space generates axisymmetric perturbations. Therefore by adopting a cylindrical reference system it is possible to combine Navier's equations and the Helmholtz potential decomposition in order to obtain the wave motion equations. Denoting the potentials (ϕ and ψ) and the vertical and radial coordinates (z and r), the governing differential equation reads:

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\frac{1}{c_p^2}\frac{\partial^2\phi}{\partial t^2} \tag{1}$$

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}-\frac{\psi}{r^2}=\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2} \tag{2}$$

where:

$$c_p=\left(\frac{\lambda+2\mu}{\rho}\right)^{1/2},\ c_s=\left(\frac{\mu}{\rho}\right)^{1/2} \tag{3}$$

while the displacement field respectively in radial and vertical downward direction is:

$$u=\frac{\partial\phi}{\partial r}-\frac{\partial\psi}{\partial z},\ w=\frac{\partial\phi}{\partial z}+\frac{1}{r}\frac{\partial(r\psi)}{\partial r} \tag{4}$$

A way of solving the differential equations (1-2) is through Fourier transform. Shifting from time domain to frequency domain reduces the problem to Bessel equations: its solution is the Bessel's function J_0 . In order to discretize the domain it is necessary to impose zero amplitude at distance $r = R$ far enough

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from the origin.

$$J_0(kR) = 0 \tag{5}$$

By choosing the constants (wavenumbers) as $k_m = \alpha_m/R$, where α_m represents the m-th root of J_0, the solution of equation (5) is achieved. The potentials in the frequency domain are consequently defined as:

$$\hat{\phi}_{mn}(r,z) = A_{mn}e^{-ik_{pzmn}z}J_0(k_m r) \tag{6}$$

$$\hat{\psi}_{mn}(r,z) = B_{mn}e^{-ik_{szmn}z}J_1(k_m r) \tag{7}$$

where:

$$k_{pzmn} = \left(\frac{\omega_n^2}{c_p^2} - k_m^2\right)^{1/2},\ k_{szmn} = \left(\frac{\omega_n^2}{c_s^2} - k_m^2\right)^{1/2} \tag{8}$$

being k_{pzmn} the wavenumber in vertical direction, k_{szmn} the wavenumber in radial direction, A_{mn} and B_{mn} constants defined by boundary conditions, J_1 the Bessel's function of the first kind.

2.2. *Discrete spectral solution*

Thanks to the linearity and homogeneity of the governing equations it is possible to use the principle of superposition. This means that by double summation over M wavenumbers and N angular frequencies ω_n one can reconstruct the whole vibration system that vanishes at $r = R$.

$$\phi_{mn}(r,z,t) = \sum_n\sum_m A_{mn}e^{-ik_{pzmn}z}J_0(k_m r)e^{-i\omega_n t} \tag{9}$$

$$\psi_{mn}(r,z,t) = \sum_n\sum_m B_{mn}e^{-ik_{szmn}z}J_1(k_m r)e^{-i\omega_n t} \tag{10}$$

Summation over N can be done by means of FFT. At this point it is straightforward to build the stiffness matrix for the layer elements from the nodal displacement obtained by (4) and the applied boundary tractions. The process needs to be done for every wavenumber and every frequency while the global stiffness matrix is assembled in the same way as in the finite element method. The final result is the following:

$$u(r,z,t) = \sum_n\sum_m \hat{u}_{mn}(z,k_m,\omega_n)\hat{F}_m J_1(k_m r)\hat{F}_n e^{i\omega_n t}$$

$$w(r,z,t) = \sum_n\sum_m \hat{w}_{mn}(z,k_m,\omega_n)\hat{F}_m J_0(k_m r)\hat{F}_n e^{i\omega_n t}$$

being u_{mn} and w_{mn} the displacements for a unit load condition, while F_m and F_n represent the Fourier-Bessel spatial coefficients and fast Fourier time coefficients of the load.

2.3. *Inverse analysis*

In order to bring the details of the proposed procedure one needs to discuss also the general framework of inverse analysis and minimization algorithm. Herein the brief explanation of main features of the inverse procedure followed by a detailed elucidation of implemented minimization algorithms is presented.
Back-calculation analysis with a particular application to constitutive model calibration is a tool widely used by many researchers (see e.g. [1, 5, 6]). In general it merges the numerically computed $\mathbf{U}_{\text{NUM}}$ and experimentally determined $\mathbf{U}_{\text{EXP}}$ measurable quantities for a discrepancy minimization. A vector of residua $\mathbf{R}$ in time t can be constructed in the following way:

$$\mathbf{R}^t(\mathbf{x}) = \mathbf{U}^t_{\text{EXP}} - \mathbf{U}^t_{\text{NUM}}(\mathbf{x}). \tag{11}$$

This measures the differences between the aforementioned measurable quantities. By adjusting the constitutive parameters (encapsulated in the vector $\mathbf{x}$) embedded in the numerical model, which in turn mimic the experimental setup, an iterative convergence towards the required solution can be achieved. The minimization of the objective function ω (within the least square frame) takes the form:

$$\omega^t = \sum_{i=1}^{n}\left(R_i^t\right)^2 = \left\|\mathbf{R}^t\right\|_2^2, \tag{12}$$

and is usually updated through the use of first-order (gradient-based) or zero-order (gradient-less) algorithms. Among the many first-order procedures that are based on either the Gauss-Newton or the steepest descent direction in a nonlinear least square methods, the Trust Region Algorithm (TRA) seems the most effective. The TRA uses a simple idea, similar to that in Levenberg-Marquardt (LM) algorithm (see e.g. [7]), which performs each new step in a direction combining the Gauss-Newton and steepest descent directions.
Here however another great tool is selected for an automatic update of the model parameters prediction, which is based on Bayes principles, namely Kalman filter. By making use of such algorithm one achieves not only the parameter estimates but also their uncertainty.

3. Concluding remarks

Herein a procedure based on the discrete spectral solution has been implemented and its verification has been obtained by comparing the results with those obtained in [3, 4]. Special consideration went into performance since code efficiency is crucial for inverse analysis algorithms. Therefore static correction technique, interpolation method of FRF and high performing libraries (LAPACK) where used. Calculations were conducted over an Intel Pentium T2330 1.60 GHz and 2 GB of RAM memory and code has proven to be efficient enough: about 0.12s for each forward computation, which is few hundred times faster then FE forward model. Additionally by adopting the Kalman filter the overall identification procedure gives a realistic perspective for very fast and robust engineering application.

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