Reliable mechanical characterization of layered pavement structures

T. Garbowski
Institute of Structural Engineering
Poznan University of Technology, Poznan, Poland.

A. Venier
Faculty of Civil and Environmental Engineering
Politecnico di Milano, Milan, Italy

ABSTRACT: Non-destructive testing is often used in pavement engineering to determine the structural status of pavements. One of the most used devices in this field is the Falling Weight Deflectometer (FWD), which produces a set of recorded surface deflections later used for parameter identification purposes. It is then important to create a proper model to interpret FWD data. In literature many authors have proposed different approaches for analysing the behavior of pavement structures. Some of them are simpler, other are more rigorous. Most commercial programs employ quasi-static analyses due to its simplicity and performance. This choice, however, may be in conflict with the purely dynamic response of the real structure. A better alternative might contemplate dynamic finite element models, which unfortunately are very costly and therefore impractical in real life applications. In this work however, the dynamic behavior of the system is described through the Spectral Element Method (SEM). Its high-performance and precision represent a valuable element in back-analysis. Besides, the introduction of different minimization algorithms (Powell's method, Levenberg-Marquardt algorithm, Extended Kalman Filter) combined with multiple starting points result in a more confident and stable solution. In conclusion, the study reports a back-calculation example for a 3-layered pavement system. The intention is to show the importance of lower frequencies and the influence of resonance phenomena in system identification.

1 INTRODUCTION

1.1 *Maintenance and rehabilitation*

Deterioration of the pavement is an inevitable process and takes place from the day after construction. In such scenario, it is mandatory to schedule a precise maintenance plan in order to ensure the full functionality of the road during its design life (figure 1). It

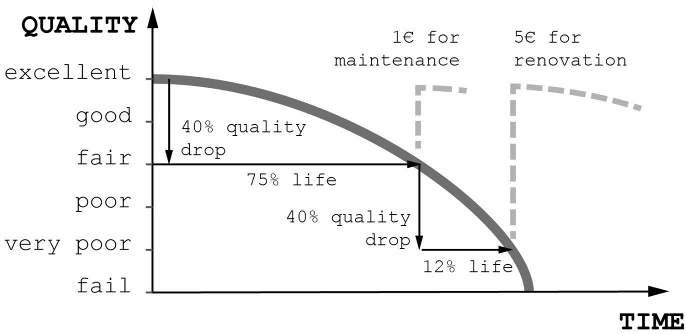

Figure 1: Pavement life management

is thus imperative to intervene in the right moment, when rehabilitation can still be conducted with lower resources. A posterior operation, dealing with structure in worse conditions, might require a different amount of finances. Fortunately, with just a limited quantity of new materials and by reemploying the existing ones, flexible pavements may be easily recycled. Nevertheless, it is still an expensive operation and, in order to plan a proper rehabilitation strategy for the whole network, it is necessary to regularly monitor the status of the roads. Such process is called pavement management and contemplates the use of specific devices for assessing the conditions of the structure by means of nondestructive testing and evaluation (NDT and NDE) methods. A typical machine used for such purpose is, in fact, the Falling Weight Deflectometer (FWD) (9, 10).

1.2 *The Falling Weight Deflectometer*

The FWD represents a precious instrument for the structural evaluation of existing flexible pavements: it produces fast and easy-repeatable tests. The basic concept behind the FWD is that the magnitude of the surface deflections caused by a given load are indirectly indicating the structural health of the system. The mechanism attempts, in fact, to replicate the effect of single heavy moving wheel acting on the pavement by applying a dynamic load. Such impulse is produced by a considerable weight and delivered to the surface by a circular plate (typically 300 mm in diameter). A rubber base at the bottom permits a uniform load pressure. The instrument is also equipped with a load-measuring cell that accounts for the exact pressure at each time interval. Several geophones (7 or 9) are located at different distances from the loading device and are responsible for rigorous displacement recording. Indeed, the sensor precision ranges between $\pm 1.0\,\mu m$ (or even $\pm 0.1\,\mu m$): accurate recording is imperative when small variations in the deflection have considerable influence on the structural response. Such deflections are then employed in a back-calculation procedure in order to assess the pavement mechanical properties. Typical tests contemplate the use of four different load levels (the entire procedure is normally completed in less than two minutes) and they are conducted every 150 meters, in order to ascertain unexpected shifts of soil and pavement properties. FWD tests are often preferred over destructive methods because they are much more rapid and do not entail the removal of pavement materials. This is a great advantage since the sought information may be available soon after the test is performed, thus saving expensive laboratory tests.

2 SPECTRAL BASED FORWARD MODEL

The proposed here procedure uses a spectral element method (SEM) instead of dynamic FE models. The main advantage, beside a substantial decrease of the computational time, is a significant increase of an experimental data to be incorporated into the inverse analysis. This is mainly because one can now freely sample the dynamic response not only in space but also in time. Moreover, since the spectral element technique makes use of the exact solution of the wave problem, a very high level of precision is guaranteed.

2.1 *Governing differential equations*

A vertical impulse load acting on a homogeneous isotropic half-space generates axisymmetric perturbations. Therefore, by adopting a cylindrical reference system, it is possible to exploit Navier's equations and the Helmholtz potential decomposition in order to obtain the wave motion equations. Denoting the potentials (ϕ and ψ) and the vertical and radial coordinates (z and r), the governing differential equation reads:

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\frac{1}{c_p^2}\frac{\partial^2\phi}{\partial t^2}$$
$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}-\frac{\psi}{r^2}=\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2} \tag{1}$$

where:

$$c_p=\left(\frac{\lambda+2\mu}{\rho}\right)^{1/2} \quad ; \quad c_s=\left(\frac{\mu}{\rho}\right)^{1/2} \tag{2}$$

while the displacement field respectively in radial and vertical downward direction is:

$$u=\frac{\partial\phi}{\partial r}-\frac{\partial\psi}{\partial z} \quad ; \quad w=\frac{\partial\phi}{\partial z}+\frac{1}{r}\frac{\partial(r\psi)}{\partial r} \tag{3}$$

A way of solving the differential equations (1) is through Fourier transform. The shifting from time domain to frequency domain reduces the equations to simple Bessel equations: its solution is the Bessel's function J_0 . In order to discretize the domain it is necessary to impose zero amplitude at distance $r=R$ far enough from the origin.

$$J_0(kR)=0 \tag{4}$$

By choosing the constants (wavenumbers) as $k_m=\alpha_m/R$, where α_m represents the m-th root of J_0, the solution of equation (4) is achieved. The potentials in the frequency domain are consequently defined as:

$$\hat{\phi}_{mn}(r,z)=A_{mn}e^{-ik_{pzmn}z}J_0(k_m r)$$
$$\hat{\psi}_{mn}(r,z)=B_{mn}e^{-ik_{szmn}z}J_1(k_m r) \tag{5}$$

where:

$$k_{pzmn}=\left(\frac{\omega_n^2}{c_p^2}-k_m^2\right)^{1/2} ;\ k_{szmn}=\left(\frac{\omega_n^2}{c_s^2}-k_m^2\right)^{1/2} \tag{6}$$

being k_{pzmn} the wavenumber in vertical direction, k_{szmn} the wavenumber in radial direction, A_{mn} and B_{mn} constants defined by boundary conditions, J_1 the Bessel's function of the first kind.

2.2 *Discrete spectral solution*

Thanks to the linearity and homogeneity of the governing equations it is possible to use the principle of superposition. This means that by double summation over M wavenumbers and N angular frequencies ω_n one can reconstruct the entire oscillating system.

$$\phi_{mn}(r,z,t)=\sum_n\sum_m A_{mn}e^{-ik_{pzmn}z}J_0(k_m r)e^{-i\omega_n t}$$

$$\psi_{mn}(r,z,t)=\sum_n\sum_m B_{mn}e^{-ik_{szmn}z}J_1(k_m r)e^{-i\omega_n t}$$

(7)

At this point, it is straightforward to build the stiffness matrix for the layer elements from the nodal displacement obtained by (3) and the applied boundary tractions. The process needs to be done for every wavenumber and every frequency while the global stiffness matrix is assembled in the same way as in the finite element method. The final displacement field is the following:

$$u(r,z,t) = \sum_n \sum_m \hat{u}_{mn}(z,k_m,\omega_n)\hat{F}_m J_1(k_m r)\hat{F}_n e^{i\omega_n t}$$
$$w(r,z,t) = \sum_n \sum_m \hat{w}_{mn}(z,k_m,\omega_n)\hat{F}_m J_0(k_m r)\hat{F}_n e^{i\omega_n t} \tag{8}$$

being $\hat{u}_{mn}$ and $\hat{w}_{mn}$ the displacements for a unit load condition, while F_m and F_n represent the Fourier-Bessel spatial coefficients and fast Fourier time coefficients of the load. Differently from FEM, however, thanks to the exactness of the found solution, one element per layer is sufficient to represent adequately the behavior of the system. More details can be found in (1, 2)

3 PARAMETER IDENTIFICATION STUDY

3.1 *Back-calculation analysis*

Back-calculation analysis with a particular application to constitutive model calibration is a tool widely used by many researchers (see e.g. (6, 4, 5, 3, 7)). In general, it involves the minimization of the discrepancy between the numerically computed $\mathbf{U}_{\text{NUM}}$ and measurable quantities $\mathbf{U}_{\text{EXP}}$. A vector of residuals $\mathbf{R}$ in frequency domain f can be constructed in the following way:

$$\mathbf{R}^f(\mathbf{x}) = \mathbf{U}^f_{\text{EXP}} - \mathbf{U}^f_{\text{NUM}}(\mathbf{x}) \tag{9}$$

By adjusting the constitutive parameters (encapsulated in the vector $\mathbf{x}$) embedded in the numerical model, which in turn mimic the experimental setup, an iterative convergence towards the required solution can be achieved. The minimization of the objective function ω (within the least square frame) takes the form:

$$min_{\mathbf{x}}\, \omega(\mathbf{x})^f = \left\|\mathbf{R}^f(\mathbf{x})\right\|_2^2 \tag{10}$$

and it is updated through the use of gradient-based algorithms (Levenberg-Marquardt (LM), Extended Kalman FIlter (EKF)) and gradient-less algorithms (Powell (P)). More details can be found in (8). Fortunately, the profound differences between each of these optimization techniques constitute the backbone of the entire program. Diverse solutions may be found by each algorithm and, among those, only the one that shows the lowest error norm is selected.

3.2 *Multistart approach*

It is known that optimization problems may be more or less dependent on the user provided initial guesses. This is due to the very nature of the objective function which is in general non-convex: the algorithm might get trapped inside a local minimum and fail to reach the optimum. There is not much to do on the algorithm side. However, as the spectral implementation is relatively not demanding, the program can afford to contemplate the coexistence of multiple initial values. By selecting the best estimate related to the lowest error norm, it is possible to eradicate the dependency on the starting guess. Thus, few more seconds of computations can produce a massive enhancement of code stability. Figure 2 documents the improvement achieved.

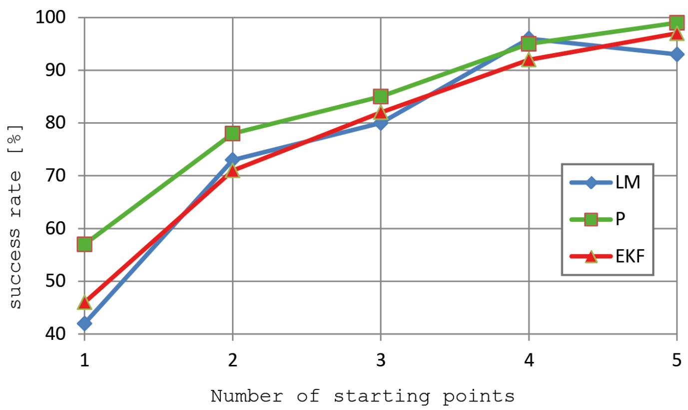

Figure 2: Success rate for different amounts of starting points

3.3 *Example: 3-layered pavement structure with rigid subbase on shallow bedrock*

The current study considers the presence of a stabilized subbase in a 3-layered system (the properties are indicated in table 3.3). This scenario represents an important practical case because a stiffer layer is typically required for pavements constructed above weak soil. Moreover, it is known that when the stiffness ratio between layers is significant, some back-calculation instabilities may arise. The here presented study is based on computer-generated deflection data.

layer	$E\,[MPa]$	$h\,[cm]$	ν	$\rho\,[Kg/m^3]$
1	2000	15	0.35	2300
2	5000	25	0.35	2000
3	25	430	0.35	1500

Table 1: Material properties.

As speculated the program strives to find a correct solution (represented by the dashed line). It can be noticed, however, that the lower frequencies (0-10Hz) are able to correctly find the stiffness values for all layers. Analyzing the following frequency range (10-20Hz), one can notice a sudden dispersion peak which

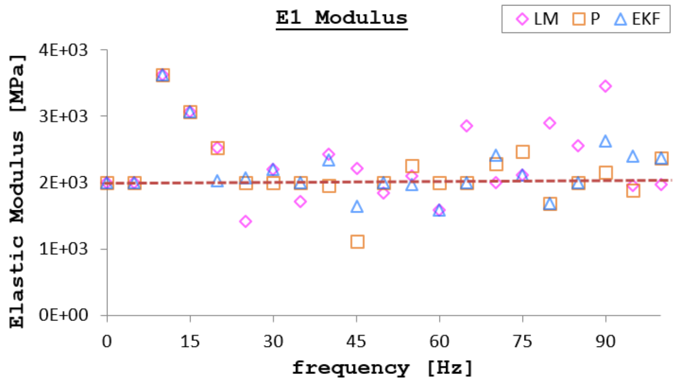

Figure 3: Dispersion representation of the first layer elastic modulus.

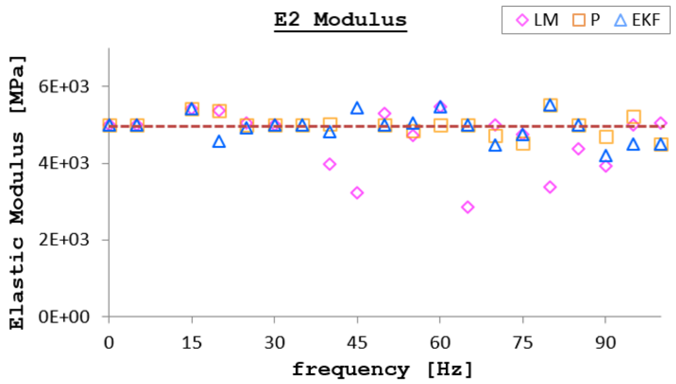

Figure 4: Dispersion representation of the second layer elastic modulus.

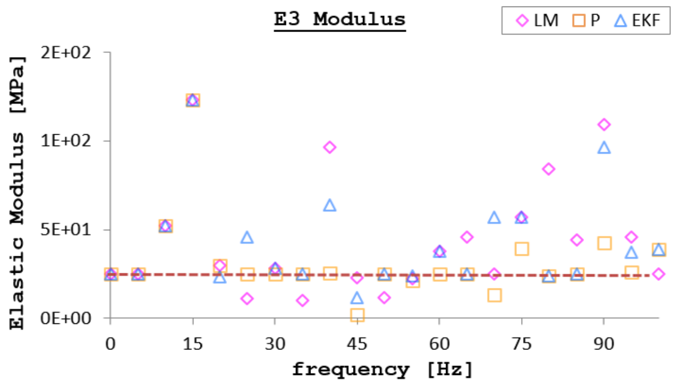

Figure 5: Dispersion representation of the third layer elastic modulus.

leads to highly incorrect results. This behavior is caused by resonance phenomena due to the shallow bedrock. Therefore, in this interval, found parameters are not to be trusted. In addition, this example shows that stiffer layers are the ones which are better identified, regardless of the depth of the layer.

4 CONCLUSIONS

Herein a procedure based on the discrete spectral solution has been implemented and its verification has been properly obtained. One of the most powerful features of SEM is the double summation approach, which avoids the typical complications rising with numerical integration between zero and infinity. Also, it was discovered that one finite spectral element is representative of the entire layer. This results in a dramatic reduction of the model complexity if compared, for instance, to FEM. Therefore, it is possible to assert that the spectral element method represents a robust and highly efficient tool for recreating the dynamic nature of the FWD test. The association of the numerical model with three distinct minimization algorithms yielded to an exhaustive back-calculation program. The program, in fact, selects only the solution with the lowest error norm, thus ensuring a better estimate. It is known, however, that search techniques are strongly conditioned by the choice of the initial guess. Here, a simple consideration, contemplating the coexistence of multiple random starting points, was able to dramatically improve the stability of the code. Later, a 3-layered example with rigid subbase and shallow bedrock was studied. It was noticed that lower frequencies usually permit more successful parameter identification. Besides, great prudence is required in the eventuality of shallow bedrock (or seasonal stiff layers). Resonance phenomena may in fact contaminate measurement and conduce to erroneous results.

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