ECCOMAS International Conference **IPM 2011** on Inverse
Problems in Mechanics of Structure and Materials
27-30 April 2011, Rzeszów–Sieniawa, Poland

STOCHASTIC ALGORITHM FOR MULTI-OBJECTIVE SHAPE OPTIMIZATION OF DENTAL IMPLANT

Tomasz Garbowski*, Krzysztof Szajek, Tomasz Łodygowski

Poznan University of Technology, Institute of Structural Engineering
ul. Piotrowo 5, 60-965 Poznan, Poland
e-mail: {tomasz.garbowski, krzysztof.szajek, tomasz.lodygowski}@put.poznan.pl

1 INTRODUCTION

The shape optimization received a lot of attention in recent years mainly due to growing interest in applications of such field in physics and engineering. The goal in geometry optimization is to find a shape among the set of all admissible shapes that minimizes a given objective function. In the present paper the stochastic programming technique based on Gaussian processes [1, 2] applied to shape optimization is presented. The optimal geometry of dental implant subjected to static loads is designed here with the objective to minimize the Mises effective stresses in selected zones of the model. The resulting stresses (state variables) depend, obviously, on boundary and loading conditions but also on geometrical parameters (control variables), which define the geometry of the model. The probable correlations between the state variables are computed by making use of Principal Component Analysis.
The proposed algorithm formulated within Bayesian framework provides additionally the information about the magnitude of correlation between state and control variables, i.e. the relevance of input-output correlation.

2 NUMERICAL MODEL

The dental implant to be optimized is a two-component system [3] consists of a root and abutment (both made of medical alloys of titanium) which are assembled by a screw. The axisymmetric geometry and linear isotropic elasticity are assumed here, mainly for model simplification. The five geometrical parameters composed of angular and linear dimensions are selected as control variables which represent the input vector $\mathbf{x}$, whereas the output vector $\mathbf{t}$ contains the maximum equivalent stress values in selected *a priori* zones. There are three zones in the model having different material properties, each zone contain three to seven sub-zones where the maximum values of Huber-Mises-Hencky (HMH) stress are recorded.

3 PROCEDURE

Usually a several thousands of training pairs have to be generated in order to approximate well such systems. An alternative (more economical) solution might be based on a stochastic surrogate model which requires a smaller number of function evaluations. In order to build such approximation we need first to collect the resulting stresses in vectors for each implant's component separately, i.e. we need to construct the input-output pairs for each zone: $\left(\mathbf{x}^i, \mathbf{t}_A^i\right)$,

$\left(\mathbf{x}^i, \mathbf{t}_B^i\right)$, $\left(\mathbf{x}^i, \mathbf{t}_C^i\right)$. Once the training pairs are constructed the probable correlation between components of the vectors $\mathbf{t}^i$ in each zone (A,B and C) needs to be ruled out of the system. An effective procedure for performing this operation is Principal Component Analysis (PCA) which: orthogonalizes the components of the vectors, it orders its orthogonal components in descending order, and it eliminates those components that contribute the least to the variation in the data set, so that the number of component in a new vectors $\mathbf{a}^i$ is smaller then in $\mathbf{t}^i$.
When the new 'uncorrelated' targets vectors $\mathbf{a}^i$ are computed the surrogate models, for each component of the vector separately, can be constructed. The very efficient way of building such approximations are Gaussian processes viewed as Bayesian linear regression over a possibly infinite number of basic functions. Assuming that a Gaussian process has zero mean, we just need to define the covariance between any two points $\mathbf{x}$ and $\mathbf{x}'$. A stationary, non-isotropic squared exponential covariance function $C\left(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}\right)$ is chosen, for which the full target covariance is given by:

$$C\left(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}\right) = \nu_0 \exp\left(-\frac{1}{2}\sum_{l=1}^{d} w_l \left(x_l^{(i)} - x_l^{(j)}\right)^2\right) + b + \sigma_\nu^2 \delta_{ij}, \tag{1}$$

where σ_ν is the variance of the target distribution, the term b represents a bias that controls the vertical offset of the Gaussian process, while ν_0 controls the vertical scale of the process. The w_l parameters allow a different distance measure for each dimension. If w_l is small then the l-th input is downweighted and have little effect on the input.

All hyperparameters in the model must be positive, so it is convenient to consider them in log space, namely: $\mathbf{h} = [\log \sigma_\nu, \log b, \log \nu_0, \log w_1, \ldots, \log w_d]$. To estimate the model parameters a supervised learning, based on a first-order batch Levenberg-Marquardt Algorithm (LMA), is used, which provides fast and robust convergence. In order to optimize the training procedure of a Gaussian process based surrogate models, the active learning criterion that improve the global model fit is also utilized (i.e. a new training points are subsequently added in the zones where the model predictions are poor).
Once the stochastic approximations of the numerical model are constructed, the prediction of the stress distribution in each zone can be computed for generic $\mathbf{x}^*$ and then the minimum of such multi-objective system can be easily computed on-line, using either first-order or gradient-free algorithm (e.g. LMA or Genetic Algorithm).

Acknowledgments: The support of Polish Ministry of Science and Higher Education under the grant R13 0020 06 is kindly acknowledged.

REFERENCES

[1] C.M. Bishop, *Pattern Recognition and Machine Learning*. Springer, 2007.
[2] C.E. Rasmussen, C.K.I. Williams, *Gaussian Processes for Machine Learning*. MIT Pres, 2006.
[3] T. Łodygowski, M. Wierszycki, K. Szajek, W. Hedzelek, R. Zagalak, Tooth-Implant Life Cycle Design, *Computer Methods in Mechanics, Lectures of the CMM 2009*, ASM 1, chapter 21, 391-422, Springer-Varlag Berlin Heidelberg 2010.