28th Nordic Seminar on Computational Mechanics
NSCM-28
A. Berezovski, K.Tamm, T.Peets (Eds.)

APPLICATION OF SPECTRAL ELEMENT METHOD TO RELIABLE MECHANICAL CHARACTERIZATION OF LAYERED PAVEMENT STRUCTURES

TOMASZ GARBOWSKI[*†] **AND ANDREA VENIER**[†]

[*]Institute of Structural Engineering
Poznan University of Technology
ul. Piotrowo 5, 60965 Poznan, Poland
e-mail: `tomasz.garbowski@put.poznan.pl`, web page:
`http://www.put.poznan.pl`

[†]Department of Civil and Environmental Engineering
Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133 Milano, Italy
e-mail: `andrea.venier@student.polimi.it` - web page:
`http://www.polimi.it`

Key words: Falling Weight Deflectomiter, Spectral Element Method, Inverse Analysis.

Summary. The subject of here presented work points out the actual and relevant issue of a health monitoring of the road structures which exhibit a permanent and progressive deterioration. The main problem arises when the critical decision concerning the rehabilitation method or/and technology of a strategic segment of the national road/highway has to be made. The crucial point, then, is to support the decision with the reliable method of structural diagnosis. Herein the diagnostic tool based on the synergic combination of the nondestructive testing method and spectral element method is presented followed by its validation through numerous examples.

1 INTRODUCTION

The health monitoring techniques of the road structures can be easily divided into two main groups: (1) destructive and (2) non-destructive.

Destructive testing is performed when more precise information regarding mechanical, physical, and chemical properties of each layer are required. Additionally, they also provide a visual inspection of pavement strata, but they are an expensive solution. In consequence, invasive tests are generally performed only when other methods might be inadequate.

On the other hand e.g. visual condition survey are useful for providing a first qualitative meter of possible deterioration. However, when conducted by persons, they may be mistakenly corrupted by the visual sensitivity of the evaluator. For this reason automatic mapping has been lately offering a better and more objective tool for detecting surface flaws. Another important group of nondestructive tests used in pavement engineering are: nuclear equipment, seismic evaluation, electromagnetic testing and deflection based. Each survey has its particular advantages and limitations, however, all are extremely useful in gathering information regarding the structural condition of the layers.

One of the most popular nondestructive test is Falling Weight Deflectometer (FWD)[6], where recorded data are indirectly used to generate an estimation of layer moduli and possibly its thicknesses. It requires (parallel to the in-situ test) an application of the analytical or numerical modeling accompanied with the inverse analysis. These additional tools are necessary because FWD provides the vertical deflections at geophone localization only while the sought quantities are usually thickness and stiffness of each pavement layer. In order to compute those values one needs to incorporate a reliable model which can mimic the behavior of the real structure and then by adjusting the embedded parameters in the model (i.e. Young's moduli, Poison's ratios, layer's thickness or bonding coefficients) the corresponding response (here the vertical displacements) can be computed. The iterative minimization of the discrepancy between measured and computed quantities can be performed either by stochastic updates of the parameters or through systematic steps in the parameter space using mathematical programming.

2 FWD DATA

Due to its crucial importance in estimating the road structural health, layer moduli backcalculation using FWD data has been subjected to enthusiastic research during the last decades. As a result, literature offers numerous examples for solving the parameter identification problem on the basis of many different approaches[6,5], spanning analytical and numerical solutions, dynamic and static formulations. The time-history data provided by the FWD apparatus represents an extraordinary resource for assessing the unknown parameters with a potentially good degree of accuracy. However, although the intrinsic dynamic nature of the test, many back-calculation programs available today rely on the elasto-static formulation, which operates on simplified testing data (maximal values of force and displacement only). This means that not only inertial and damping contributions, which play a fundamental role in this type of problem, are neglected, but also a great amount of the acquired information is discarded.

Since programs based on the elasto-static approach may produce inaccurate solutions, some disputable adjustment factors are introduced in order to compensate such intrinsic flaws[5]. Also critical engineering judgment is required in order to properly interpret those results. Even if most static analyses employed in parameter identification problems are performed by means of the multilayer elastic theory, some routines also operate with the method of equivalent thicknesses (based on Boussinesq half-space elastic theory) or the finite element method.

In theory, the finite element method is a powerful and versatile tool for conducting full dynamic investigations. However, back-calculation procedures demand a certain computational speed that finite element formulation cannot guarantee. Fortunately, there is another tool which is able to keep the dynamic precision of the finite element method and the good performance of the multilayer elastic theory: the Spectral

Element Method (SEM).

3 NUMERICAL MODEL

The main features of the SEM are the principle of superposition and the discrete Fourier transform theory: the dynamic solution is obtained by superimposing the behavior of a finite amount of wave modes at multiple discrete frequencies. Afterwards, time-domain response is computed by means of FFT.

The domain is meshed in a very simple manner: one element is in fact sufficient for representing an entire layer with constant properties. Moreover, the spectral approach assembles the stiffness matrix in the same way as for the FEM. The difference, however, is that such matrix is full, exact, and defined for each discrete frequency and wavenumber. Such properties produce therefore accurate solutions with modest computational effort: precision and efficiency together.

In this scenario the rapidity of spectral analysis becomes an extremely valuable resource: data collected by the FWD test accompanied with spectral element method and inverse analysis are making the ideal combination for parameter identification problems. A more detailed description of the SEM mathematical formulation both in forward and backward frame can be found in[2,3].

Lastly, it is necessary to remind that the exactness of the formulation is possible only when wave solutions are available (mostly one-dimensional problems). Also, the superposition principle, which represents the core of the whole method, only holds for linear system whose properties are constant over time.

4 EXAMPLES

All of the examples computed here are generated by the SEM model accompanied with Powel (P)[4], Levenberg-Marquardt (LM)[4] or Extended Kalman Filter (EKF)[1] minimization algorithm. The reference data used here is pseudo-experimental meaning it is generated by a numerical model. In the inverse procedure each analysis is formulated in frame of the least squares minimization, where the objective function is casted as a difference between the reference and computed displacement amplitudes in frequency domain (after FFT transform). The results are considered as correct when the set of sought parameters converged to the exact values (the ones embedded in the reference pseudo-experimental model). Each inverse analysis consists of 5 minimization procedures, i.e. each starts from 5 different stochastically generated points in the parameters space. The final solution, then, is the one with the smallest objective function value.

Many practical examples were taken into consideration, each representing different complexity - from 3- to 5-layered structures with and without bedrock, with and without artificial noise. As a representative example and corresponding results of the whole investigation presented here correspond to the model with 3 layers. The reference data is incorporated with different level of Gauss noise.

The results showed in the table 1 represent the error percentage of inverse analysis for different noise levels. This shows two main features of the investigation: (1) lower frequencies used in the construction of the objective function give in general better results, (2) noise higher than 3% affects the frequency response to the extent that none of the minimization algorithms could correctly identify the system.

	error[%]		
defl.Noise [%]	**freq.0**	**freq.5**	**freq.10**
0.5	2	4	18
1	2	4	19
3	-	-	-

Table 1: Noise results error expressed as % of the correct solution

5 CONCLUSIONS

Summarizing, spectral element method, as expected, represents a robust and efficient tool for reproducing the dynamic nature of the FWD test. As a matter of fact, once the measured displacements are converted into the frequency domain by means of Fourier's transform (FFT), the objective function may be defined through frequency components rather than displacements. Besides, the presence of three different optimization algorithms running in parallel, reinforces the validity of the final result.

REFERENCES

[1] J.W. Choi, R. Wu, J.M. Pestana and J. Harvey, New Layer-Moduli Back-Calculation Method Based on the Constrained Extended Kalman Filter *Journal of Transportation Engineering* **136**, 20–30, (2010).

[2] R. Al-Khoury, A. Scarpas, C. Kasbergen and J. Blaauwendraad, Spectral element technique for efficient parameter identification of layered media. I. Forward calculation, *International Journal of Solids and Structures* , **38**, 1605–1623, (2001).

[3] R. Al-Khoury, A. Scarpas, C. Kasbergen and J. Blaauwendraad, Spectral element technique for efficient parameter identification of layered media. II. Inverse calculation, *International Journal of Solids and Structures* , **38**, 8753–8772, (2001).

[4] J. Nocedal and S.J. Wright, **Numerical Optimization**, Springer, (2006).

[5] T.M. Westover and B.B. Guzina Engineering framework for the self-consistent analysis of FWD data, *Transp. Research Record*, **2005**, 55–63, (2007).

[6] S.M. Zaghloul, T. D. White, V. P. Drnevich, and B. Coree, Dynamic analysis of FWD loading and pavement response using a three dimensional dynamic finite wlement program *American Society for Testing and Materials*, 125–138, (1998).