

HOMOGENIZATION OF CORRUGATED BOARDS THROUGH INVERSE ANALYSIS

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Abstract. *Homogenization of corrugated cardboards plays an important role in computer aided design of cardboard packages. The numerical analysis performed on a full structural model gives very precise results however is computational costly and therefore impractical in industrial applications. Thanks to adopted model simplifications an engineer responsible for a new design of a box can make it easily and reliably on a homogenized structure. The only remaining though very important issue is to find out how to compute a set of effective parameters in simplified model in order to keep its behavior qualitatively and quantitatively close to the response of a full structural model in a wide range of loading conditions. Here a calibration method which take a benefit from both simple experimental tests and numerical homogenization techniques of thin periodic plates is presented. A numerical or analytical homogenization of elastic properties of paperboard is well documented in the literature however the calibration of the effective properties in an inelastic region is still an open subject. Authors discuss herein a synergic combination of basic cardboards laboratory tests, its numerical simulations and inverse analysis as a tool for a full calibration of a simplified cardboard model. A sensitivity analysis and a simplified case study on a different experimental setups is also presented here and results are reported.*

1 INTRODUCTION

1.1 Paperboard overview

Paper is one of the most popular material exploited in our everyday life, commonly used in e.g. aseptic food, trade and transport packaging or printing industries. It is produced from cellulose pulp which is softened either mechanically or chemically. Paperboard consists of few paper layers glued together with a starch glue. In a production process the paperboard becomes thicker and stiffer, which allows it to be more flexible and universal material. One of the great feats of paper and paperboard is its recyclability both in terms of recovery of a raw material and savings of an energy. Every item produced with paper can be easily reshaped again into cellulose pulp and converted to a new material. Recovered raw material has a limited usability for example in food packaging industry but has very wide application in paper packaging industry. This characteristic is the reason why paper packagings are so cheap and in the same time environmental friendly thus so widely used. In paper industry paperboard due to its low bending stiffness limited by its thickness is either assembled in much stiffer structure called corrugated board or used as wrappings, book covers, etc.

Corrugated cardboard consists of few flat paperboard layers called liners, distanced by sine-shaped layers called flutings. Number of liners in corrugated cardboard (later on referred as to cardboard) ranges from 1 (in liner-fluting system usually serving as cushion wrapping) to 4 (in heavy industrial paperboard bins), while number of flutings ranges from 1 to 3 respectively. The most common form of cardboard is two-walled cardboard consisting of printable superior liner, fluting and interior liner. Such structure can bear relatively high loads which allows boxes to be stacked in warehouses.

Recently new type of cardboard boxes ('shelf ready box') was introduced which is designed in such a way that it can be easily opened by tearing off a top part of a box and than directly placed on shop's shelves. In order to produce tearing a perforation is introduced which reduces bearing-capacity of boxes. Designing of such perforated boxes is tedious experimental work thus there is a lot of space for numerical analyses which can automate and improve this process. There are many problems that must be coped with during numerical simulations of cardboard behavior, e.g. complicated geometry, interlayer debonding, elasto-plastic constitutive modeling, damage initiation and evolution, to name just a few.

Paper and paperboard's microscopic structure is determined by industrial manufacturing processes. During the paper ply production the cellulose fibers tend to align themselves along a main direction in which machine is moving material referred as to Machine Direction (MD). This direction is typically of greatest strength and stiffness though lowest ductility. The other in-plane direction is usually referred as to Cross Direction (CD) and out-of-plane is called thickness direction or simply ZD. Due to fibrous structure of paper and directional alignment of fibers the macroscopic yield of paperboard is different in tension than in compression. This is due to structural response of fibers (buckling), thus compression yield is usually lower than tension yield. Therefore constitutive modeling of paperboard must account for orthotropy (both in elastic and inelastic phase) but also should include tension-compression distinction in plastic phase. Among most common plasticity models used for paperboard modeling one can name Hill [14], Hoffman [15], Tsai-Wu [23], Makela [20], Ristinmaa [13]. There are more sophisticated models e.g. Xia et. al [24] but the more complicated the model the more parameters must be determined experimentally [12] making the identification process more complex. Common characteristic of mentioned models is orthotropic yield criterion. Number of inelastic parameters needed to be determined by experiments ranges from 7 in Hill, through 10 in Tsai-Wu to 27 in Xia et. al.

Typically paperboard's mechanical properties are obtained through Short-Span Compression Test (SCT). In this test sample which length is 0.7 mm is compressed and stress-strain curve is obtained. To determine all parameters (stiffness moduli, yield stresses, hardening parameters) in every orthotropic direction a set of tests on three different samples are required - MD (0 degree), CD (90 degree) and 45 degree sample. On the other hand, cardboard properties can be also obtained from Edge Crush Test (ECT). This test involves a compression of a cardboard sample rather than separate plies of paperboard (as in previous test) and also provides stress-strain curves which are used in an inverse procedure to identify elasto-plastic effective parameters.

Due to geometrical characteristic paperboard is usually modeled as shell elements in Finite Elements Methods (FEM). As fluting period is very small compared to cardboard structures (ranging from 6-8 mm while structures ranges from centimeters to few meters in boxes) structural model of cardboard requires fine mesh thus very high computational power to be evaluated. In order to cope with this researchers adopted homogenization methods that replaces structural models with a single-layered shell model, which provided proper effective parameters, can produce accurate results for much lower computational cost. In the literature proposed homogenization techniques for paperboard usually deal only with elastic properties [2, 4, 6, 7, 22] while for full description of cardboard mechanics also plasticity must be taken into consideration.

This work deals with finding effective elasto-plastic parameters for shell model that replaces structural cardboard model through mixed numerical-experimental approach. Standard paperboard and cardboard experimental tests are used, namely SCT and ECT, to calibrate effective parameters by inverse analysis in a frame of least square error fit of numerical and experimental data.

2 METHODS

2.1 Material model

In this work paperboard is modeled using orthotropic Hooke's law combined with Hill yield criterion and linear hardening. Material is described in elastic phase with 9 independent constants: $E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, \nu_{12}, \nu_{13}, \nu_{23}$ (reduced to just four in plane stress conditions), where 1,2,3 are three perpendicular directions (in case of paperboard MD, CD, ZD, respectively). Constitutive relationship is formulated as:

$$\boldsymbol{\varepsilon} = \mathbf{C}\boldsymbol{\sigma}, \quad (1)$$

where $\boldsymbol{\varepsilon} = [\varepsilon_{11}, \varepsilon_{22}, 2\varepsilon_{12}]^T$ is a strain vector, $\mathbf{C}$ is compliance matrix, and $\boldsymbol{\sigma} = [\sigma_{11}, \sigma_{22}, \sigma_{12}]^T$ is a stress. For plain stress and shell elements orthotropic compliance matrix can be written as:

$$\mathbf{C} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}, \quad (2)$$

When stress reaches some threshold value material starts to yield. Yield criterion generally can be formulated as:

$$\Phi(\boldsymbol{\sigma}, \kappa) = f(\boldsymbol{\sigma}) - [\sigma_y(\kappa)]^2 \leq 0, \quad (3)$$

where κ is a state variable and $f(\boldsymbol{\sigma})$ is a function which depends on a stress state and its characteristic is given for a particular yield criterion. Material yields when $\Phi = 0$ and it is in elastic state when $\Phi < 0$. Hill yield criterion is often used in modeling of paperboard's plasticity [5, 16] (though its origin is in metal plasticity) and is formulated in the following way:

$$f(\boldsymbol{\sigma}) = F_1(\sigma_{11} - \sigma_{22})^2 + F_2(\sigma_{22} - \sigma_{33})^2 + F_3(\sigma_{11} - \sigma_{33})^2 + F_4\sigma_{12}^2 + F_5\sigma_{23}^2 + F_6\sigma_{13}^2 \quad (4)$$

where σ_{ij} is a stress in element's local coordinate system which is aligned with material's direction. Coefficients F_i defining equation (4) are formulated with yield stresses in given direction σ_{ij}^0 and reference yield σ_0 .

$$\begin{aligned} F_1 &= \frac{\sigma_0^2}{2} \left(\frac{1}{(\sigma_{11}^0)^2} + \frac{1}{(\sigma_{22}^0)^2} - \frac{1}{(\sigma_{33}^0)^2} \right), \\ F_2 &= \frac{\sigma_0^2}{2} \left(\frac{-1}{(\sigma_{11}^0)^2} + \frac{1}{(\sigma_{22}^0)^2} + \frac{1}{(\sigma_{33}^0)^2} \right), \\ F_3 &= \frac{\sigma_0^2}{2} \left(\frac{1}{(\sigma_{11}^0)^2} - \frac{1}{(\sigma_{22}^0)^2} + \frac{1}{(\sigma_{33}^0)^2} \right), \\ F_4 &= \frac{\sigma_0^2}{(\sigma_{12}^0)^2}, \\ F_5 &= \frac{\sigma_0^2}{(\sigma_{13}^0)^2}, \\ F_6 &= \frac{\sigma_0^2}{(\sigma_{23}^0)^2}, \end{aligned} \quad (5)$$

As plane stress elements (thus also shells) do not provide transverse shear stresses (in thin plates theory) coefficients F_5, F_6 are irrelevant and can be omitted. Typically for practical use and formulation, yield stresses are replaced with corresponding plastic potentials:

$$R_{ij} = \frac{\sigma_{ij}^0}{\sigma_0}$$

Important note to take when using this criterion is fact that in main stresses space this criterion is in general elliptical cylinder but in some cases depending on values of yield stresses in different directions, it can degenerate into hyperbolic surface, thus not provide description of physical material [3, 8].

Hardening model was assumed to be linear, depending solely on effective plastic strain. It can be mathematically formulated as:

$$\sigma_y = \sigma_0 + H\bar{\varepsilon}^p, \quad (6)$$

where H is hardening modulus, σ_0 initial yield in reference direction, σ_y current yield stress and $\bar{\varepsilon}^p$ is equivalent plastic strain, which can be calculated during integration of elasto-plastic laws with equation:

$$\bar{\varepsilon}^p = \int \frac{\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p}{\sigma_y} dt, \quad (7)$$

This model provide a simplified approximation of real paperboard behavior, it does not distinguish between tension and compression but for sake of modeling of ECT it is sufficient enough.

2.2 Modeling ECT experiment

Edge Crush Test is one of mostly used tests that characterize cardboard. Numerically it can be simulated in order to obtain cardboard structural response to compressive loads provided cardboard constituents' parameters (liners and flutings) were obtained in SCT. Numerical scheme consists of 3 parts - two rigid plates that transmit loads to the specimen and cardboard sample which in-plane size is 100x25 mm. For this experiment friction interaction between plates and cardboard was used to model boundary conditions of system. Bottom rigid plate is fixed so it serves as support for sample and top plate is pushed into direction of sample for given displacement u which is big enough to trigger plastic response. Moreover, displacement of top plate is constrained in other directions so that sample does not undergo additional twisting or bending. In order to obtain accurate results non-linear analysis is performed with maximal step equal to 0.02% of total applied displacement. Mesh size is set to approximate 0.5 mm which provide fine mesh to model fluting's geometry.

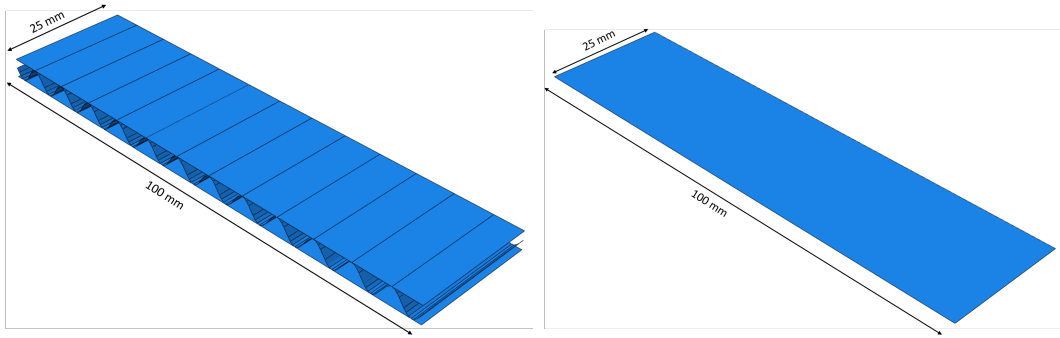


Figure 1: Numerical Edge Crush Test Scheme. Structural and homogenized models.

Due to fact that such model consists of roughly 190,000 degrees of freedom it can not be effectively used to simulate larger cardboard structures. In order to reduce problem size thus increase applicability of cardboard in numerical analyses one can model it using single shell element which has such properties that it's response (force against displacement) is equivalent to the one obtained for example in ECT - such process is called homogenization. As a result model of equivalent stiffness can be obtained which number of degrees of freedom is roughly 2000 (for 3 mm element size), that is almost 100 times less in comparison to full structural model.

2.3 Homogenization in elastic regime

The goal of homogenization is to obtain obtain stiffness of heterogeneous system and then replace it with homogeneous system of equivalent stiffness. In case of cardboard it can be done by means of Classical Laminate Plate Theory (CLPT). According to Kirchhoff's assumptions strains in plate and shell elements can be decomposed into strains of mid-plane ϵ and curvature effects η :

$$\epsilon = \epsilon_m + z\eta. \quad (8)$$

In-plane conditions stresses can be then described with an equation:

$$\boldsymbol{\sigma} = \mathbf{Q} (\boldsymbol{\varepsilon}_m + z\boldsymbol{\eta}), \quad (9)$$

where $\mathbf{Q}$ is a material stiffness matrix obtained as an inverse of compliance matrix from equation (2). Internal forces are then obtained using their definitions:

$$\mathbf{N} = \int \boldsymbol{\sigma} dz = \int \mathbf{Q} (\boldsymbol{\varepsilon}_m + z\boldsymbol{\eta}) dz = \mathbf{A}\boldsymbol{\varepsilon}_m + \mathbf{B}\boldsymbol{\eta}, \quad (10)$$

$$\mathbf{M} = \int \boldsymbol{\sigma} z dz = \int \mathbf{Q} (\boldsymbol{\varepsilon}_m + z\boldsymbol{\eta}) z dz = \mathbf{B}\boldsymbol{\varepsilon}_m + \mathbf{D}\boldsymbol{\eta},$$

Thus shell elements stiffness that transforms strains (linear and curvatures) into internal forces can be written as:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\eta} \end{bmatrix}. \quad (11)$$

CLPT allows to calculate **ABD** matrix by integration of cardboard section's stiffness according to formulas:

$$\mathbf{A}(x) = \int \mathbf{Q} dz = t_{ls} \mathbf{Q}_{ls} + t_f \mathbf{Q}_f(\theta) + t_{li} \mathbf{Q}_{li},$$

$$\mathbf{B}(x) = \int \mathbf{Q} z dz = t_{ls} \mathbf{Q}_{ls} z_{ls} + t_f \mathbf{Q}_f(\theta) z_f + t_{li} \mathbf{Q}_{li} z_{li}, \quad (12)$$

$$\mathbf{D}(x) = \int \mathbf{Q} z^2 dz = \mathbf{Q}_{ls} \left(z_{ls}^2 t_{ls} + \frac{t_{ls}^3}{12} \right) + \mathbf{Q}_f(\theta) \left(z_f^2 t_f + \frac{t_f^3}{12} \right) + \mathbf{Q}_{li} \left(z_{li}^2 t_{li} + \frac{t_{li}^3}{12} \right),$$

where subscripts ls , li and f denote superior liner, interior liner and fluting respectively.

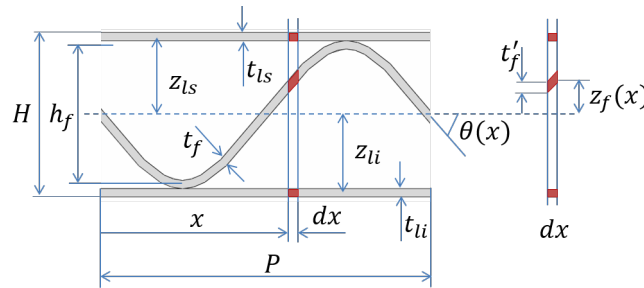


Figure 2: Cardboard section with characteristic CLPT properties annotated.

It is vital to understand that due to fluting shape it's material parameters for each section differs in global coordinate system (as local system is constantly rotating). Thus for each section dx fluting parameters must be rotated. If fluting's position is described with sine function:

$$h(x) = \frac{h_f}{2} \sin \left(\frac{2\pi x}{P} \right),$$

where: h_f is a distance between liners and P is a fluting's period, then rotation angle is given by:

$$\theta(x) = \arctan \left(\frac{dh(x)}{dx} \right), \quad (13)$$

Total stiffness of section is now obtained by integrating section's stiffness over interval of fluting's period:

$$\begin{aligned} \mathbf{A}_h &= \frac{1}{P} \int_0^P \mathbf{A}(x) dx, \\ \mathbf{B}_h &= \frac{1}{P} \int_0^P \mathbf{B}(x) dx, \\ \mathbf{D}_h &= \frac{1}{P} \int_0^P \mathbf{D}(x) dx, \end{aligned} \quad (14)$$

Having homogeneous stiffness one can obtain material stiffness matrix either from equations (15),(16):

$$\mathbf{Q}_h = \frac{\mathbf{A}_h}{t_h}, \quad (15)$$

$$\mathbf{Q}_h = \frac{12\mathbf{D}_h}{t_h^3}, \quad (16)$$

where t_h is taken in such a way that both matrices $\mathbf{A}_h$ and $\mathbf{D}_h$ can be equivalent to cardboard ones. It was suggested [1] that good approximation of this thickness is given by:

$$t_h \approx \sqrt{12 \frac{D_{11} + D_{22} + D_{33}}{A_{11} + A_{22} + A_{33}}}. \quad (17)$$

This method though simple and straight-forward to implement provide good homogenization in elastic region. Unfortunately it can not provide effective plastic parameters thus must be enhanced with other tools for full homogenized cardboard description. CLPT can serve as starting point for optimization algorithms greatly reducing number of iterations needed to find parameters giving best fit to experimental data.

2.4 Inverse analysis

The inverse analysis (also known as back-calculation analysis) is widely used by many researchers ([25, 18, 11, 12, 9, 10, 17, 19]) for constitutive model calibration. It merges the numerically computed $\mathbf{U}_{\text{NUM}}$ and experimentally determined $\mathbf{U}_{\text{EXP}}$ measurable quantities for discrepancy minimization. A vector of residua $\mathbf{R}$ can be constructed in the following way:

$$\mathbf{R} = \mathbf{U}_{\text{EXP}} - \mathbf{U}_{\text{NUM}}(\mathbf{x}). \quad (18)$$

This measures the differences between the aforementioned measurable quantities. By adjusting the constitutive parameters (encapsulated in the vector $\mathbf{x}$) embedded in the numerical model,

which in turn mimic the experimental setup, an iterative convergence towards the required solution can be achieved. The minimization of the objective function ω (within the least square frame) takes the form:

$$\omega = \sum_{i=1}^n (R_i)^2 = \|\mathbf{R}\|_2^2, \quad (19)$$

and is usually updated through the use of first-order (gradient-based) or zero-order (gradient-less) algorithms. Procedures based on a soft method (e.g. genetic algorithms, simulated annealing, particle swarm algorithms) can be also used for function minimization, especially when the function is non-convex and so has many local minima. However, such algorithms usually require many iterations. Among the many first-order procedures that are based on either the Gauss-Newton or the steepest descent direction in a nonlinear least square methods, the Trust Region Algorithm (TRA) seems the most effective. The TRA uses a simple idea, similar to that in Levenberg-Marquardt (LM) algorithm (see e.g. [21]), which performs each new step in a direction combining the Gauss-Newton and steepest descent directions. The LM algorithm computes new directions using the following formula:

$$\Delta \mathbf{x} = -(\mathbf{M}_{\mathbf{x}} + \lambda \mathbf{I})^{-1} \mathbf{g}_{\mathbf{x}}, \quad (20)$$

where λ is an internal parameter, $\mathbf{g}_{\mathbf{x}} = \nabla \omega(\mathbf{x})$ is the gradient of the objective function ω with respect to the parameters $\mathbf{x}$

$$\mathbf{g}_{\mathbf{x}} = \frac{\partial \omega}{\partial \mathbf{x}}, \quad (21)$$

and the Hessian $\mathbf{M}_{\mathbf{x}} = \nabla^2 \omega(\mathbf{x})$ is a second partial derivative of ω with respect to the parameters $\mathbf{x}$:

$$\mathbf{M}_{\mathbf{x}} = \frac{\partial^2 \omega}{\partial \mathbf{x}^2}, \quad (22)$$

In the nonlinear least square approach method, the gradient and Hessian matrix can be computed using the Jacobian matrix:

$$\mathbf{J}(\mathbf{x}) = \frac{\partial \mathbf{R}}{\partial \mathbf{x}}, \quad (23)$$

so the gradient and Hessian matrix are defined, respectively:

$$\mathbf{g}(\mathbf{x}) = \mathbf{J}^T \mathbf{R}, \quad \mathbf{M}(\mathbf{x}) \simeq \mathbf{J}^T \mathbf{J}. \quad (24)$$

Such approximation of the Hessian, which can be computed ‘for free’ once the Jacobian is available, represents a distinctive feature of least squares problems. This approximation is valid if the residuals are small, meaning we are close to the solution. Therefore some techniques may be required in order to ensure that the Hessian matrix is semi-positive defined (see e.g. [21]).

One of the main issues of the trust region approach, which to a large extent determines the success and the performance of this algorithm, is in deciding how large the trusted region should be. Allowing it to be too large can cause the algorithm to face the same problem as the classical Newton direction line search, when the model function minimizer is quite distinct from the minimizer of the actual objective function. On the other hand using too small a region means that the algorithm will miss the opportunity to take a step substantial enough to move it much closer to the solution.

Each k -th step in the trust region algorithm is obtained by solving the sub-problem defined by:

$$\min_{\mathbf{d}_k} m_k(\mathbf{d}_k) = f(\mathbf{x}_k) + \mathbf{d}_k^T \nabla f(\mathbf{x}_k) + \frac{1}{2} \mathbf{d}_k^T \nabla^2 f(\mathbf{x}_k) \mathbf{d}_k, \quad \|\mathbf{d}_k\| \leq \Delta_k \quad (25)$$

where Δ_k is the trust region radius. By writing the unknown direction as a linear combination of Newton and steepest descent direction, the sub-problem will take the following form:

$$\begin{aligned} \min m_k(\mathbf{x}_k) = & f(\mathbf{x}_k) + [s_1 \mathbf{d}_k^{SD} + s_2 \mathbf{d}_k^N]^T \nabla f(\mathbf{x}_k) + \\ & + \frac{1}{2} [s_1 \mathbf{d}_k^{SD} + s_2 \mathbf{d}_k^N]^T \nabla^2 f(\mathbf{p}_k) [s_1 \mathbf{d}_k^{SD} + s_2 \mathbf{d}_k^N], \end{aligned} \quad (26)$$

under the constraints:

$$\|s_1 \mathbf{d}_k^{SD} + s_2 \mathbf{d}_k^N\| \leq \Delta_k. \quad (27)$$

The problem now becomes two-dimensional and it is solved for the unknown coefficients s_1 and s_2 . In order to find both s_1 and s_2 in eq. (27) a set of nonlinear equations can be solved using, for example, the Newton-Raphson techniques. Herein this approach is implemented using an inverse procedure to compute the discrepancy minimization between the reaction force measured in full structural model and the corresponding one computed by the homogenized numerical model.

3 CALIBRATION

In order to test and evaluate the proposed method a numerical pseudo-experiment was designed. Homogeneous sample of known material parameters and orientations were compressed by applying displacement and measuring reaction force. In order to simulate real experiment's uncertainty an additional random uniformly distributed noise was introduced to the numerical measurements within a range of ± 3 [N]. The three simulations were conducted on different samples: CD sample, 45-degree sample and combined CD and 45-degree samples (both evaluated at one run of algorithm). The goal of this test was to evaluate if randomly generated starting points will converge to values of reference parameters within few iterations of algorithm. Additionally such test helped establishing which numerical test is best for identification of certain parameters, for example E_2 is accurately recognized in CD test and combined CD, 45 degree test.

As seen in figure 3 algorithm gives convergence to the reference parameters with limited accuracy. The best results are obtained in combined test with ν_{12} locked where only E_1, G_{12} are not converging to original values. This is caused by so called compensation (or ill-posedness) and is directly related to the mechanics of orthotropic materials. In 45-degrees test directional elastic modulus (E_{45}) depends on 4 parameters: $E_1, E_2, \nu_{12}, G_{12}$ according to a formula:

$$\frac{1}{E_{45}} = \frac{1 - \nu_{12}}{4E_1} + \frac{1 - \nu_{21}}{4E_2} + \frac{1}{4G_{12}},$$

As a result an algorithm do not converge to reference parameters but instead finds any combination that provides accurate E_{45} . As combined test uses also CD test, E_2 is accurately found through this experiment thus either ν_{12}, E_1 should be locked (as they form together one term that affects E_{45}) or G_{12} alone. Having this problem in mind one can assess that algorithm performs well, convergence to reference values of plastic parameters in combined test and can be used to identify effective parameters.

In order to enhance understanding of mechanics in each directions sensitivity analysis was performed on CD and 45-degrees samples. Sensitivity analysis is technique which provides information how model reacts to change of each input's value. This process can further show which parameters are active and recognizable in each test. Reference model was taken from previous test. Each of input parameters was disturbed by 5% and change of force-displacement

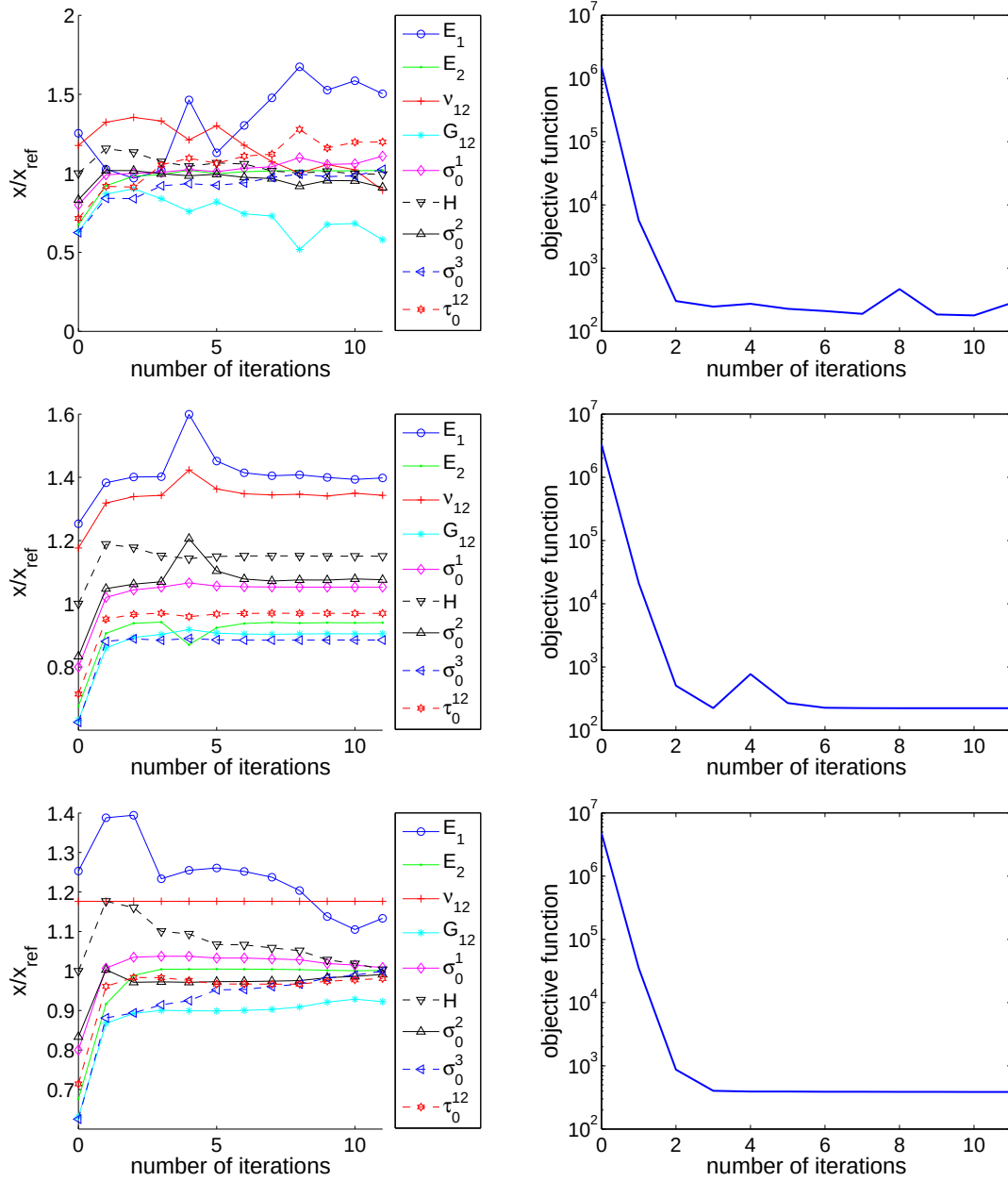


Figure 3: Convergence of initial parameters to reference values in various test types.

path was measured. For each test force was read in fixed 20 points and sensitivity was measured with formula:

$$s_i = \sum_{i=1}^{20} \left(F_i^{ref} - F_i \right)^2, \quad (28)$$

Results acquired in sensitivity analysis are presented in figure 4 .

Figure 4 clearly shows that most important parameters in CD is initial yield stress in CD, σ_0^{22} . Other parameters that influence this tests are E_2 and H . This proves that in CD test only those three factors can be evaluated. On the other hand, in 45-degree there are more then just three active parameters. Similarly to CD test the yield stresses ($\sigma_0^3, \sigma_0^{12}$) have greatest influence on model. Additionally elastic parameters that are important in this tests are E_2, G_{12} .

This experiments leads to conclusion that most of parameters can be evaluated in CD and

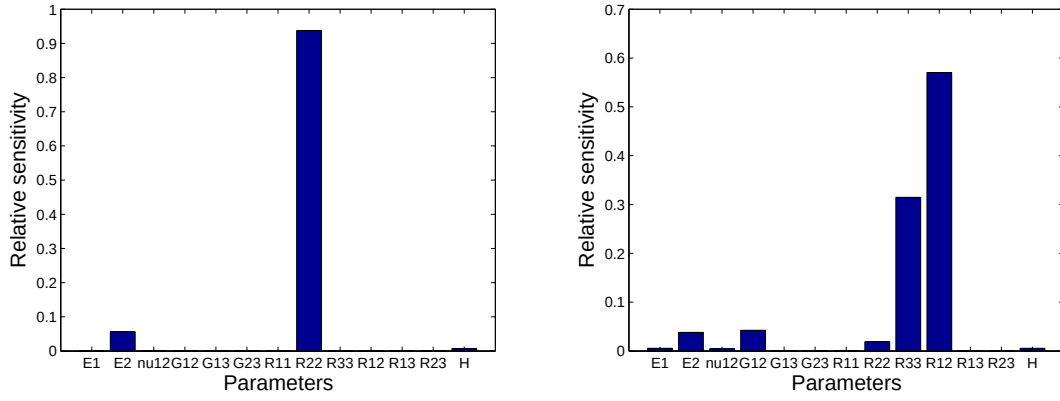


Figure 4: Sensitivity of homogenized ECT model to material parameters.

45-degrees tests but both must be used, either separately or combined.

4 RESULTS

Method proposed in chapter 2 was implemented in Abaqus/Standard. Cardboard that was evaluated is designated with code KLSKL595C and it's properties are presented in tables 1, 2.

Parameters	Top Liner	Fluting	Bottom Liner
t [mm]	0.29	0.3	0.29
E_1 [MPa]	3326	2614	3326
E_2 [MPa]	1694	1532	1694
ν_{12} [-]	0.34	0.32	0.34
G_{12} [MPa]	859	724	859
σ_0 [MPa]	15	15	15
R_{11} [-]	1	1	1
R_{22} [-]	0.5	0.5	0.5
R_{33} [-]	0.5	0.5	0.5
R_{12} [-]	0.4	0.4	0.4
H [MPa]	500	500	500

Table 1: Material Parameters of KLSKL595C cardboard.

Wavelength of fluting [mm]	Height of fluting [mm]
8	3.5

Table 2: Geometric Parameters of KLSKL595C cardboard.

Model of such cardboard was built in Abaqus/Standard consisting of roughly 190,000 degrees of freedom. Cardboard size was taken according to ECT standards - 100x25 mm. Sample was loaded with displacement equal to 0.2 mm which results in 1% average strains. Non-linear analysis was performed in order to capture wrinkling of liners and fluting near loading rigid

plates. Such reference experiment was performed for samples cut in 4 different orientations, namely: CD (0 deg), 30 deg, 45 deg and 60 deg rotations with respect to CD. Force vs displacement plots for ECT tests are presented in figure 5.

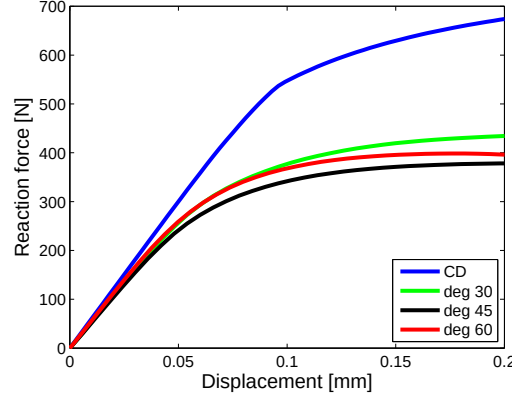


Figure 5: Force versus displacement plots for numerical ECT test on structural samples

Figure 5 clearly shows that displacement applied to sample induces non-linear response which is influenced both by non-linear geometry and plasticity of paperboard.

In order to find effective parameters of homogenized sample two steps were taken. Firstly initial guess is to be found. Elastic parameters were calculated using CLPT and results of this analyses are presented in table 3. Number of integration points for single period of cardboard was taken as 1000 which provides accurate result.

Parameters	Value
t [mm]	5.317
E_1 [MPa]	372.02
E_2 [MPa]	294.96
ν_{12} [-]	0.218
G_{12} [MPa]	94.89

Table 3: Effective parameters of KLSKL595C cardboard obtained with CLPT.

Initial plastic properties were estimated and are given in tables for each experiment.

Function that was to be optimized was constructed from difference between ECT structural tests and ECT homogenized tests. For each of experiment force was measured and then interpolated for fixed set of 20 points along displacement path. Function was then constructed as relative error of homogenized sample's response to structural response:

$$R_i = 1 - \frac{F_i^{\text{hom}}}{F_i^{\text{str}}}, \quad (29)$$

As a second step of obtaining effective parameters function given in (29) was optimized using algorithms presented in chapter 2. This step was performed for few sets of experiments, namely: CD alone to obtain effective E_2, s_0, H while all others parameters were set constant. Results

from this experiment were used as new initial guesses in procedure performed on samples oriented in 30 deg, 45 deg, 60 deg directions. This results in 3 sets of effective parameters obtained in 4 runs of experiments. In order to prevent parameters compensation E_1 and ν_{12} were set constant. Another approach was taken which bases on conducting 2 experiments simultaneously and optimization function consists then of 2×20 points. Results are presented in tables 4, 5.

Parameters	Initial	Optimal CD	Optimal 30 deg	Optimal 45 deg	Optimal 60 deg
E_1 [MPa]	372.02	372.02	390.61	390.62	390.62
E_2 [MPa]	294.97	280.21	280.21	280.21	280.21
ν_{12} [-]	0.218	0.218	0.229	0.229	0.229
G_{12} [MPa]	94.90	94.90	54.65	65.28	79.03
σ_0 [MPa]	1.34	1.00	1.00	1.00	1.00
R_{11} [-]	2	2	1.716	1.552	1.552
R_{22} [-]	1	1	1	1	1
R_{33} [-]	1	1	0.443	0.583	0.451
R_{12} [-]	1	1	0.738	0.486	0.696
H [MPa]	50	76.03	76.03	76.03	76.03
LSQE	0.532	0.011	0.0087	0.0158	0.0196

Table 4: Inverse analysis of all 4 tests conducted separately.

Parameters	Initial	CD+30 deg	CD+45 deg	CD+60 deg
E_1	372.02	372.85	370.61	365.81
E_2	294.97	280.35	280.68	280.21
ν_{12}	0.218	0.213	0.211	0.216
G_{12}	94.90	56.13	49.40	74.94
σ_0	1.34	1.02	1.02	1.03
R_{11}	2	1.22	1.26	1.27
R_{22}	1	1	1	1
R_{33}	1	0.609	0.567	0.574
R_{12}	1	0.518	0.522	0.544
H	50	68.32	49.40	46.60
LSQE	13.90	0.018	0.025	0.027

Table 5: Inverse analysis of two combined tests.

Clearly results obtained by first method variant (CD test to obtain 3 parameters rest are obtained from supplementary tests) are not consistent. Although, from table 4 it can be concluded that each test indeed reduces greatly least square error of fit, obtained parameters between different supplementary tests differ in great range (for e.g. R_{12} ranges from 0.486 to 0.738). Some of those discrepancies can be attributed to parameters compensation (for e.g. R_{12} , R_{33}) but more likely proposed variant is flawed.

On the other hand, combined tests gives much better consistency between different tests combinations. Most of elastic parameters (E_1 , E_2 , ν_{12}) are very close both to initial value and

together. This trend is not matched only by G_{12} which differs from initial value and between tests. This suggests that CLPT method provides accurate approximation to most of elastic parameters, but overestimates G_{12} by about factor of 2. Inconsistency of G_{12} value's between test can be contributed to structural response of sample. The more sample is rotated from CD sample, the less fluting actually carries load, but simultaneously material parameters rotate with respect to loading direction which contributes to non-linear trend that cannot be captured by simple shell element. In this variant of method, plastic potentials are very close together usually differing by less than 5% from average value which is very satisfactory accuracy provided that least-square error (LSQE) is reduced greatly in each test. Hardening modulus H , although close in two tests, differs almost by 50% in third test. This error can be explained as follows. From figure 5 it can be concluded that for each test hardening modulus is slightly different. CD test presents highest value (estimated as 76.03 - refer to table 4), followed by 30 deg test, 45 degree test and 60 degree test having least steep slope. During combined test algorithm will try to minimise error from this difference, hence it will find some middle value. If so, combined CD+30 degrees test will have greatest value, followed by CD+45 degree test and CD+60 degrees test will have least value of effective hardening modulus, this is indeed trend presented in table 5.

5 CONCLUSIONS

Due to rapid changes in the market of paper packaging industry, the new sophisticated numerical method should be employed in the design process of cardboard boxes. This can dramatically simplify and automate the prototyping of nontraditional boxes, provided the numerical analysis is fast and robust. In order to speed up the computations the effective elasto-plastic parameters of corrugated paperboards should be computed and used in the homogenized model of cardboard. Therefore the appropriate homogenization of structural model of cardboard together with reliable identification of effective parameters are important issues in the design process of corrugated paperboards boxes.

This work deals with a homogenization procedure which takes into account non-linear effects in cardboard. In the literature a few proposals of homogenization, limited to elastic properties of corrugated board only, can be found, however the homogenization of inelastic parameters is still an open subject. Here the elasto-plastic effective parameters (embedded in an orthotropic elasticity combined with the generalized Hills model) are computed through mixed numerical-experimental method. The set of experiments of paperboard: short compression test (SCT) and cardboard: edge crash test (ECT) are used to calibrate effective parameters of homogenized system in elasto-plastic regime of loading. The Trust Region (TR) algorithm was used to obtain best-fit match of numerical and experimental data through least square method. Obtained parameters show good agreement with experimental data.

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