



Transient bending amplification during lowering of reinforced concrete sunk wells under stratified soil and groundwater effects

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Abstract

Reinforced concrete sunk wells are frequently used in general, industrial, and water-engineering construction. However, the sinking process presents an engineering challenge due to the complex interaction between the well structure and heterogeneous soil, particularly in the presence of groundwater. Under normal conditions, a circular well behaves as a thick-walled cylinder under uniform compression, but for larger diameters or during tilting, the effects of bending must also be considered. During sinking, it acts as a tube with free ends, and after constructing the bottom plug, it transforms into a cylindrical tank. Therefore, the analysis should account for the spatial behavior of the structure and its changing static conditions. Traditional analytical and empirical methods often fail to capture the variability of stresses and uplift forces during the sinking process. This study presents a finite element analysis of reinforced concrete sunk well under different soil conditions. A custom-built axisymmetric FEM model was developed in MATLAB to simulate the gradual lowering of a well with a diameter of 6 m, a depth of 6 m, and wall thickness of 30 cm. The model included analytically defined lateral earth pressure, buoyant uplift, and frictional resistance at the well–soil interface. The results showed that the most unfavorable stress states occur in stratified soils with soft-plastic layers and a high groundwater level, while dense non-cohesive soils provide the most favorable stress distribution. The study confirms the effectiveness of the FEM method in designing safe and reliable sunk well structures.

Keywords Axisymmetric modeling · Finite element analysis · Groundwater uplift · Lateral earth pressure · Soil–structure interaction · Sunk well

1 Introduction

Reinforced concrete sunk wells remain one of the fundamental solutions in civil and environmental engineering for underground infrastructure such as stormwater systems, pump stations, deep foundations, and tunnels [1–7]. They are particularly advantageous where open excavation is impractical due to spatial constraints, groundwater conditions, or weak soil layers [8–11]. The construction process involves progressive sinking under self-weight combined with internal excavation [1, 2, 4, 6], resulting in a complex interaction between the well structure and the surrounding soil [8, 12–21].

Under service conditions, a circular sunk well behaves as a thick-walled cylinder subjected primarily to radial compression [22–24]. However, when larger diameters or construction-stage effects are considered, bending and stability phenomena must also be addressed [25–29]. Studies on cylindrical shells, tanks, and silos have demonstrated strong sensitivity to geometric imperfections, boundary conditions, and nonlinear material effects [26–35]. Importantly, the structural scheme of a sunk well changes during its life cycle: during lowering, it behaves as a cylindrical shell with free edges, whereas after bottom plug construction it transforms into a free-supported tank system [1–3, 22, 36–40]. This change in static scheme suggests that the most unfavorable internal forces may arise during construction rather than in the final design configuration.

Geotechnical conditions play a decisive role in the structural response during sinking [8–10, 14–19, 42]. In homogeneous dense soils, stress distributions tend to be relatively uniform and stable [8, 9, 43, 44]. In contrast, stratified

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deposits with alternating stiff and soft layers may lead to stress concentrations, differential movements, and localized instability [9, 10, 21]. Field observations and monitoring campaigns have confirmed the importance of construction control and soil migration mechanisms in deep caisson behavior [6, 7, 10]. The presence of groundwater further complicates the response by generating buoyant uplift forces and modifying effective stress conditions [8, 10, 21, 43], resulting in a coupled system of lateral earth pressure, hydrostatic action, and frictional resistance along the well–soil interface [12–21].

Traditional design approaches rely on analytical shell theory [22–24] combined with simplified subsoil representations such as the Winkler foundation model or elastic half-space assumptions [12–20, 22, 42, 44]. More advanced analyses employ elastic–perfectly plastic constitutive models, including Mohr–Coulomb and Drucker–Prager formulations [17, 46–48], while cohesive soils may require hardening models such as Cam Clay for improved realism [8, 17, 37, 45–49]. Recent research emphasizes refined soil–structure interaction modeling and calibration of subgrade parameters [20, 21, 43, 44, 50–52], as well as nonlinear response and stability assessment of cylindrical reinforced concrete shells [23, 24, 48–61].

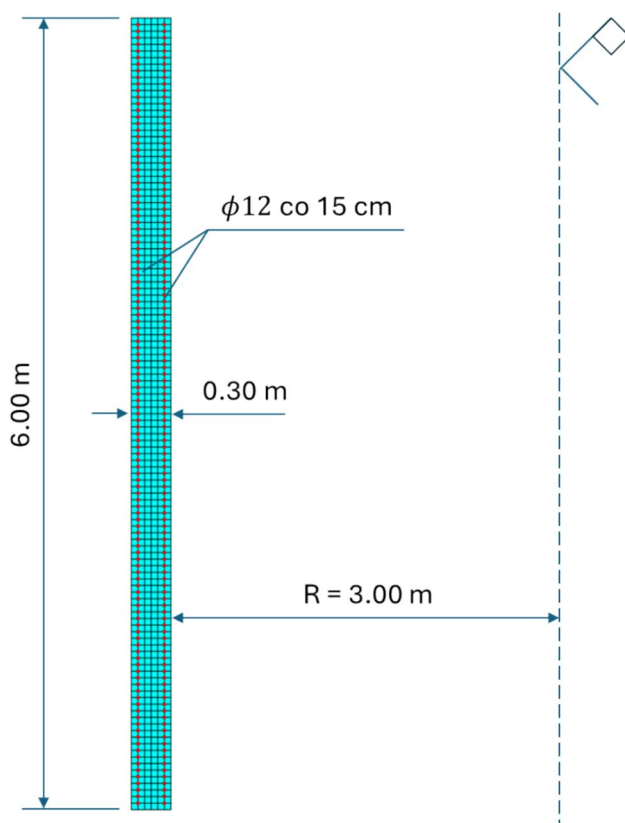


Fig. 1 The cross-section through the well wall

Despite this progress, relatively limited attention has been devoted to systematic quantification of stage-dependent internal-force evolution in sunk wells during the lowering process [1–5, 21, 37–40, 45, 50, 53, 54]. In practice, wells are typically designed for their final embedded state, even though the construction phase involves evolving boundary conditions and depth-dependent loading.

The objective of the present study is to perform a comparative finite element analysis of reinforced concrete sunk wells subjected to varying soil stratification and groundwater conditions during stepwise lowering [20, 21, 41, 45, 50–52]. A custom-built axisymmetric FEM framework implemented in MATLAB is employed to simulate depth-updated lateral earth pressure, hydrostatic action, and sidewall friction [8, 12–21]. Forty geotechnical–hydrogeological variants are analyzed to quantify bending-moment amplification (MAF) relative to the conventional post-plug design configuration and to identify critical embedment stages governed by stiffness contrast and effective stress reduction.

The novelty of the study lies in three aspects. First, the lowering process is simulated explicitly in a stage-by-stage manner with depth-dependent updating of external actions. Second, the analysis covers 40 soil–groundwater configurations in a consistent comparative framework. Third, the transient construction states are quantified directly against the final conventional design configuration. By isolating the amplification mechanisms associated with stratification and groundwater, the proposed model provides a computationally efficient and mechanically transparent tool for assessing construction-stage internal-force demand in sunk wells.

2 Methods and materials

2.1 Geometry and structural configuration

The analyzed structure is a reinforced concrete sunk well representative of typical stormwater and pumping-station applications. The geometry is axisymmetric: an internal diameter of 6.0 m, wall thickness of 0.30 m, and total height of 6.0 m (Fig. 1). The bottom edge is modeled as a cutting-edge enabling penetration into the ground.

The wall is treated as a monolithic reinforced concrete section. A typical reinforcement layout is assumed ($\phi 12$ mm at 150 mm spacing in both vertical and circumferential directions). The reinforcement is not modeled discretely. Instead, its contribution is incorporated at the sectional level by defining equivalent membrane and bending stiffnesses of the reinforced concrete wall. This homogenization preserves the global structural response within a linear-elastic framework while avoiding explicit bar discretization.

Rather than introducing a single effective Young's modulus, the reinforced section is treated as a composite cross-section, and its stiffness is represented through equivalent sectional rigidities consistent with classical reinforced concrete theory. This approach ensures that: (a) global membrane stiffness; (b) global bending stiffness; and (c) coupling between axial and bending effects are consistently represented at the element level, while local cracking and nonlinear reinforcement behavior remain outside the scope of the present comparative study. Consequently, the adopted stiffness model is suitable for global linear-elastic comparison of stage-dependent structural response, but it is not intended to reproduce stiffness degradation associated with concrete cracking, reinforcement yielding, or other nonlinear sectional effects.

Concrete is assumed as C30/37 with Young's modulus $E_c = 33$ GPa, Poisson's ratio $\nu = 0.2$, and density $\rho = 2500$ kg/m³. Reinforcing steel is assumed as B500 with $E_s = 200$ GPa and yield stress $f_y = 500$ MPa.

2.2 Governing assumptions and axisymmetric formulation

The sinking process is modeled as a sequence of quasi-static equilibrium states under axisymmetric geometry and circumferentially uniform external actions. The following assumptions are adopted:

- axisymmetric geometry and loading with respect to the well axis,
- linear-elastic isotropic response of the homogenized wall section,
- soil not modeled as a continuum; its influence introduced through analytically prescribed depth-dependent actions applied at the well boundaries,
- small displacements and linear kinematics,
- staged lowering represented by discrete embedment increments, each solved as an independent equilibrium problem.

Under these assumptions, the problem is expressed in cylindrical coordinates (r, z, θ) with $\partial(\cdot)/\partial\theta = 0$.

The adopted representation of soil actions as analytically prescribed boundary loads is a deliberate simplification introduced to isolate the structural consequences of changing embedment, stratification, and groundwater level within a consistent comparative framework. Accordingly, the model should be interpreted as a mechanism-oriented engineering formulation intended for relative assessment of construction-stage bending demand, rather than as a full predictive simulation of site-specific soil deformation.

2.3 Kinematics

The displacement field is described by radial and vertical components:

$$u_r(r, z), u_z(r, z). \quad (1)$$

The corresponding axisymmetric strain–displacement relations are:

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \varepsilon_z = \frac{\partial u_z}{\partial z}, \varepsilon_\theta = \frac{u_r}{r}, \gamma_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \quad (2)$$

In matrix form:

$$\varepsilon = [B] u_e, \quad (3)$$

where $[B]$ is the strain–displacement matrix derived from element shape functions and u_e is the nodal displacement vector.

2.4 Constitutive relations

For isotropic linear elasticity under axisymmetric conditions, the stress–strain law is:

$$\begin{bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{bmatrix} = [D] \begin{bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{bmatrix} \quad (4)$$

where $[D]$ is the constitutive matrix. For isotropic linear elasticity under axisymmetric conditions, the constitutive matrix $[D]$ takes the form:

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}, \quad (5)$$

where E and ν denote the effective sectional elastic modulus and Poisson's ratio adopted for the homogenized reinforced concrete wall. In the present study, the stiffness parameters correspond to the equivalent linear-elastic response of the reinforced concrete section, calibrated to preserve its global membrane and bending rigidity.

2.5 Finite element discretization and boundary conditions

A custom axisymmetric finite element (FE) model was implemented in MATLAB. The wall domain is discretized using four-node axisymmetric isoparametric elements with

two translational degrees of freedom per node (u_r, u_z). The formulation captures the coupled membrane–bending response of the cylindrical wall under depth-dependent radial actions while retaining computational efficiency for parametric runs.

The element stiffness matrix is obtained from the weak form of equilibrium:

$$[K_e] = \int_V [B]^T [D] [B] dV. \quad (6)$$

For axisymmetry, $dV = 2\pi r dr dz$, leading to:

$$[K_e] = 2\pi \int_{A_e} [B]^T [D] [B] r dr dz. \quad (7)$$

The global equilibrium system is assembled in standard FE manner:

$$[K]u = F. \quad (8)$$

Boundary conditions are selected to prevent rigid-body motion while representing the construction-stage structural scheme (open shaft). The top edge is kept free, reflecting the real condition during lowering. The bottom edge is constrained only to the extent needed for numerical stability (elimination of rigid modes) while allowing radial deformation consistent with external soil actions. A separate reference configuration was used for the final design-state evaluation. In that case, the well was analyzed in its fully embedded post-plug condition, which differs from the open-shaft lowering stages by the stabilizing effect of the base closure and the corresponding redistribution of internal forces. This final configuration serves only as the normalization state for the amplification metric and as the conventional design reference for comparison with transient lowering stages.

Mesh density is chosen based on preliminary convergence checks. A discretization of 6 elements through thickness and 120 elements along the wall height provided stable internal-force predictions with negligible change upon further refinement.

2.6 Depth-dependent external actions (soil and groundwater)

Since the soil continuum is not explicitly modeled, its influence is introduced through analytically prescribed depth-dependent actions applied as distributed pressures/tractions along the outer wall and at the base.

The lateral earth pressure (active condition) is defined as:

$$\sigma_h = K_a \gamma h, \quad (9)$$

where K_a is the active earth pressure coefficient, γ is the soil unit weight, and h is embedment depth.

Hydrostatic pressure due to groundwater is:

$$p_w = \gamma_w h_w, \quad (10)$$

where γ_w is the unit weight of water and h_w is the water head measured from the ground surface (according to scenarios A–D).

Sidewall frictional resistance is introduced through:

$$F_f = \mu \sigma_v A_{contact}, \quad (11)$$

where μ is the friction coefficient at the well–soil interface, σ_v is the vertical stress at depth, and $A_{contact}$ is the contact area associated with a given boundary segment.

At each lowering stage, the distributions σ_h , p_w , and frictional effects are updated consistently with the current embedment depth and soil-layer arrangement. The corresponding equivalent nodal forces are obtained by numerical integration of boundary tractions and incorporated into the global load vector:

$$[K]u = F(h), \quad (12)$$

where $F(h)$ denotes the stage-dependent loading state.

Because the surrounding ground is not modeled as a continuum, the formulation does not capture soil arching, displacement-driven stress redistribution, or contact-pressure evolution governed by local compatibility conditions. These effects may be important in layered deposits and in situations involving pronounced stiffness contrast. The present study therefore focuses on the structural response to prescribed stage-dependent actions and uses this simplified framework to identify relative amplification mechanisms rather than to reproduce the full coupled soil–structure interaction problem.

2.7 Soil profiles and parametric variants

The mechanical response of the sunk well was analyzed for several soil profiles representing a range of field conditions. The parameters were selected based on typical values found in the Polish and European geotechnical practice for medium-dense soils and verified against the literature [1–3].

General assumptions:

- Total well depth: 6 m
- Number of soil layers: 1 to 3,
- Layer thickness: 1–6 m.

Soil types:

- S1—dense coarse sand ($I_D=0.7$),
- S2—medium-dense coarse sand ($I_D=0.5$),
- S3—loose coarse sand ($I_D=0.3$),
- C1—plastic clay ($I_L=0.4$),
- C2—soft plastic clay ($I_L=0.6$),
- O—peat (organic soil) ($I_L=1.5$).

Groundwater levels:

- A—no groundwater (dry conditions),

- B—groundwater table at 4 m depth,
- C—groundwater table at 2 m depth,
- D—groundwater table at the surface (0 m depth).

The analysis covers 40 geotechnical–hydrogeological variants (Table 1), including 1–3 soil layers and groundwater scenarios A–D. Soil types include three sands (S1–S3), two clays (C1–C2), and organic soil (O), with parameters summarized in Table 2. This configuration enables systematic comparison of (i) homogeneous vs stratified conditions; and (ii) dry vs groundwater-affected states under the same structural model and staged lowering protocol.

Table 2 presents the geotechnical parameter values for the six soil types adopted for the calculations.

Figure 2 illustrates representative depth-dependent external actions adopted in the engineering formulation, including the combined lateral earth pressure and hydrostatic pressure (blue) and the sidewall frictional resistance (red). Ground layer boundaries and groundwater level are indicated by dashed lines.

2.8 Definition of stage-dependent amplification metrics

To quantify the influence of construction-stage effects, a bending moment amplification factor (MAF) is introduced as:

$$MAF(h) = \frac{M(h)}{M_{design}}, \quad (13)$$

where $M(h)$ denotes the maximum bending moment obtained at embedment depth h , and M_{design} corresponds to the maximum bending moment in the conventional post-plug design configuration. In the present study, M_{design} refers to the fully embedded post-plug reference configuration conventionally adopted for structural design. This reference state corresponds to the final embedment depth, with the bottom plug in place and external actions defined consistently with the final soil and groundwater conditions of a given variant. For each geotechnical–hydrogeological variant, the peak amplification over all stages is defined as:

$$MAF_{max} = \max_h \left(\frac{M(h)}{M_{design}} \right). \quad (14)$$

The embedment depth at which MAF_{max} occurs is denoted as the critical depth h_{crit} . These quantities are used in the parametric evaluation presented in Sect. 3.

Table 1 40 geotechnical–hydrogeological variants adopted for the calculations

No.	Layer arrangement (from top)	Water variant
1	S1 (6 m)	A
2	S2 (6 m)	A
3	S3 (6 m)	A
4	C1 (6 m)	A
5	C2 (6 m)	A
6	S1 (6 m)	B
7	S2 (6 m)	B
8	C1 (6 m)	B
9	C2 (6 m)	B
10	O (6 m)	C
11	S1 (3 m)/C1 (3 m)	A
12	S2 (3 m)/C1 (3 m)	B
13	C1 (2 m)/S2 (4 m)	C
14	C2 (2 m)/S3 (4 m)	D
15	S3 (3 m)/C2 (3 m)	A
16	S2 (2 m)/C2 (4 m)	C
17	O (2 m)/C2 (4 m)	D
18	O (1 m)/S3 (2 m)/C2 (3 m)	D
19	S1 (2 m)/C1 (2 m)/S2 (2 m)	B
20	C1 (3 m)/S3 (3 m)	C
21	S2 (3 m)/C1 (2 m)/S1 (1 m)	A
22	S3 (2 m)/O (2 m)/C2 (2 m)	D
23	S1 (4 m)/C1 (2 m)	C
24	C2 (3 m)/S2 (3 m)	B
25	S1 (1 m)/C2 (5 m)	C
26	C1 (2 m)/S3 (2 m)/C2 (2 m)	D
27	S2 (3 m)/S1 (3 m)	A
28	C1 (2 m)/C2 (4 m)	B
29	S3 (1 m)/O (2 m)/C2 (3 m)	D
30	S2 (2 m)/S1 (2 m)/C1 (2 m)	C
31	C1 (4 m)/S3 (2 m)	A
32	O (3 m)/C2 (3 m)	D
33	S1 (2 m)/S2 (2 m)/S3 (2 m)	B
34	S3 (3 m)/C1 (1 m)/S2 (2 m)	B
35	C1 (3 m)/C2 (3 m)	C
36	S2 (3 m)/C2 (3 m)	C
37	S3 (2 m)/O (1 m)/C2 (3 m)	D
38	S1 (4 m)/S2 (2 m)	A
39	S1 (2 m)/C1 (4 m)	B
40	S2 (1 m)/C2 (2 m)/O (3 m)	D

Table 2 The geotechnical parameter values for the six soil types adopted for the calculations [1, 42, 62–65]

Names	Soil types	E [MPa]	γ [kN/m ³]	ϕ [°]	c [kPa]	I_D	I_L
S1	Dense coarse sand	70	18.0 (sw) 19.0 (w) 20.5 (vw)	38	0	0.7	–
S2	Medium-dense coarse sand	50	17.0 (sw) 18.0 (w) 20.0 (vw)	35	0	0.5	–
S3	Loose coarse sand	20	16.5 (sw) 18.0 (w) 19.5 (vw)	32	0	0.3	–
C1	Plastic clay	15	20.5	8	35	–	0.4
C2	Soft plastic clay	8	19.5	5	15	–	0.6

where: E—modulus of elasticity (Young's modulus), γ —volumetric weight, ϕ —angle of internal friction, c—cohesion, I_D —density index (applicable to non-cohesive soils), I_L —plasticity index (applicable to cohesive soils), (sw)—slightly wet, (w)—wet, (vw)—very wet

2.9 Stepwise lowering procedure and MATLAB implementation

Lowering is modeled incrementally in 12 steps of 0.5 m, representing successive embedment positions. Each increment is solved as a quasi-static equilibrium state. At every stage, the solver computes:

- displacements (u_r, u_z),
- distributions of axial force N , bending moment M , and shear force Q along the wall height,
- principal stress fields and tensile/compressive zones,
- frictional reaction and base reaction components used to assess equilibrium and uplift balance.

All stage results are stored to track the evolution of internal forces and to identify critical embedment ranges, particularly at transitions between soil layers and under elevated groundwater conditions.

The staged analysis is executed by an outer loop over embedment increments and an inner assembly/solution procedure based on Eqs. (7)–(8). For each stage h_i , the algorithm:

1. constructs the external load vector $F(h_i)$ using Eqs. (9)–(12) and the current soil-layer/water configuration,
2. assembles the global stiffness matrix $[K]$ from element contributions (Eq. (7)),
3. solves $[K]u = F(h_i)$ (Eq. (12)),
4. post-processes stresses via Eq. (4)–(5) and internal forces along the wall,
5. stores stage outputs for subsequent parametric comparison and MAF extraction (Eqs. (13)–(14)).

Verification included both mesh refinement checks and consistency comparison with simplified analytical reference cases for cylindrical shells under hydrostatic and linearly

varying radial pressure. These checks confirmed the correctness of the axisymmetric formulation, numerical stability of the adopted discretization and the robustness of the staged loading routines.

In addition to mesh refinement checks, the implementation was verified against simplified reference cases for axisymmetric cylindrical walls subjected to hydrostatic and linearly varying radial pressure distributions. The comparison was intended to confirm the consistency of displacement and internal-force trends, the correct assembly of axisymmetric stiffness contributions, and the stability of the staged loading procedure. Owing to the comparative and parametric character of the present study, the verification was focused on numerical consistency and trend agreement rather than on reproducing a full benchmark problem with continuum soil modeling.

The numerical procedure is fully described in the manuscript. The MATLAB implementation used in this study constitutes the authors' research software and is not publicly distributed. However, selected implementation details, input datasets for the 40 geotechnical–hydrogeological variants, and representative verification cases can be provided upon reasonable request for academic purposes.

3 Results and discussion

3.1 Stage-dependent evolution of bending moments

The staged finite element simulations enabled continuous tracking of internal force evolution during progressive embedment of the sunk well. For each of the 40 geotechnical–hydrogeological variants, bending moments were evaluated at 12 embedment depths between 0.5 and 6.0 m. The evolution of bending moment distribution during lowering is illustrated for a representative layered configuration

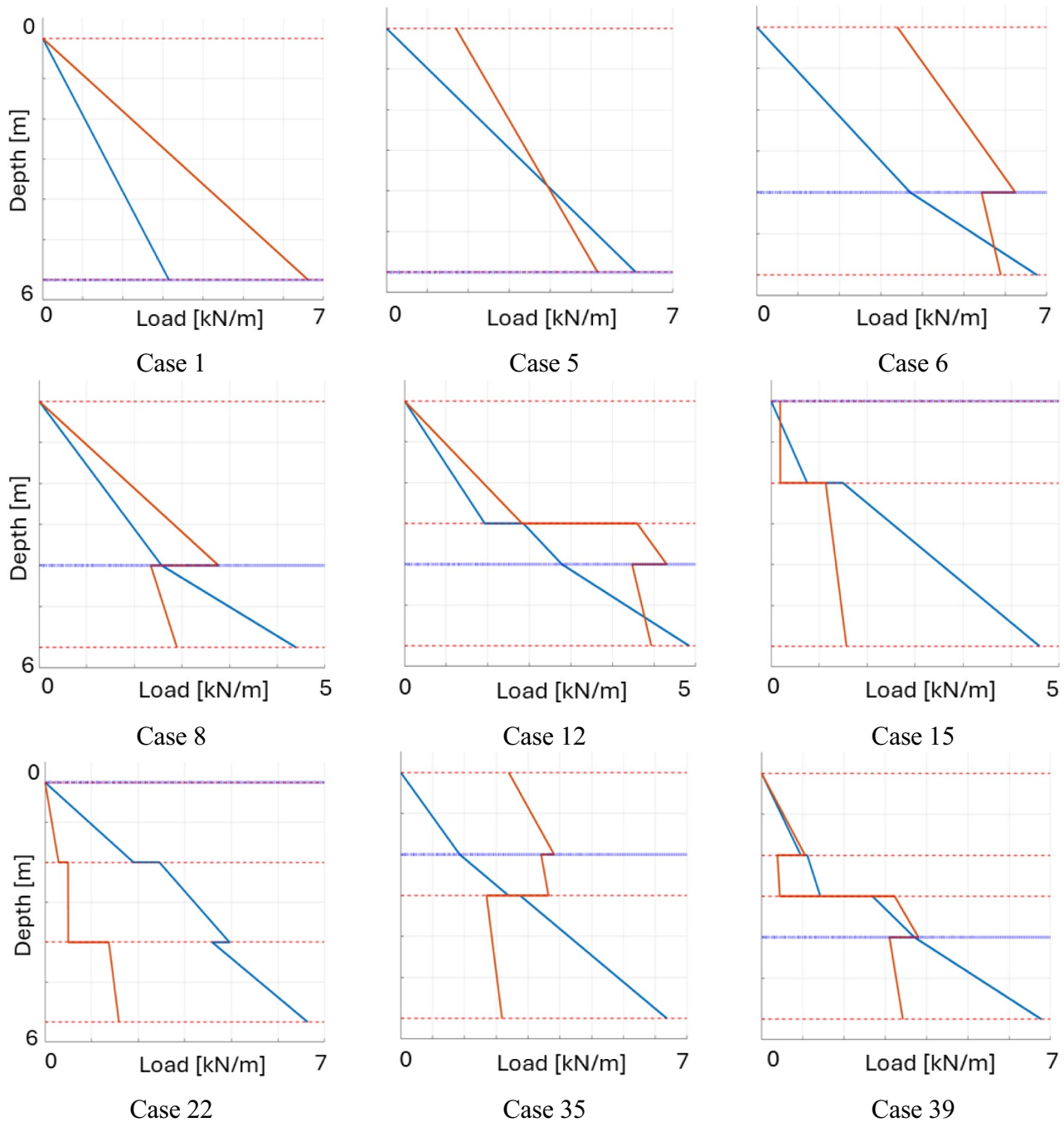


Fig. 2 Representative depth-dependent loading schemes used in the parametric study. The blue line denotes the total radial action composed of lateral earth pressure and hydrostatic pressure, while the red

line denotes sidewall frictional resistance. Dashed horizontal lines indicate soil-layer boundaries and groundwater level

in Fig. 3, where moment diagrams are shown for selected embedment depths together with the fully embedded post-plug reference state conventionally used for design.

Figure 3 clearly demonstrates that the location and magnitude of maximum bending moment change during lowering. In intermediate stages, peak tensile demand shifts toward the lower third of the wall, while in the final configuration the distribution becomes more uniform and reduced

in magnitude. In several variants, a local sign reversal between lowering and final state is observed.

While Fig. 3 illustrates the behavior for a single representative configuration, Fig. 4 extends this comparison by showing the evolution of maximum bending moment values across selected soil-groundwater scenarios.

Figure 4 confirms that the most unfavorable bending demand often occurs at intermediate embedment depths

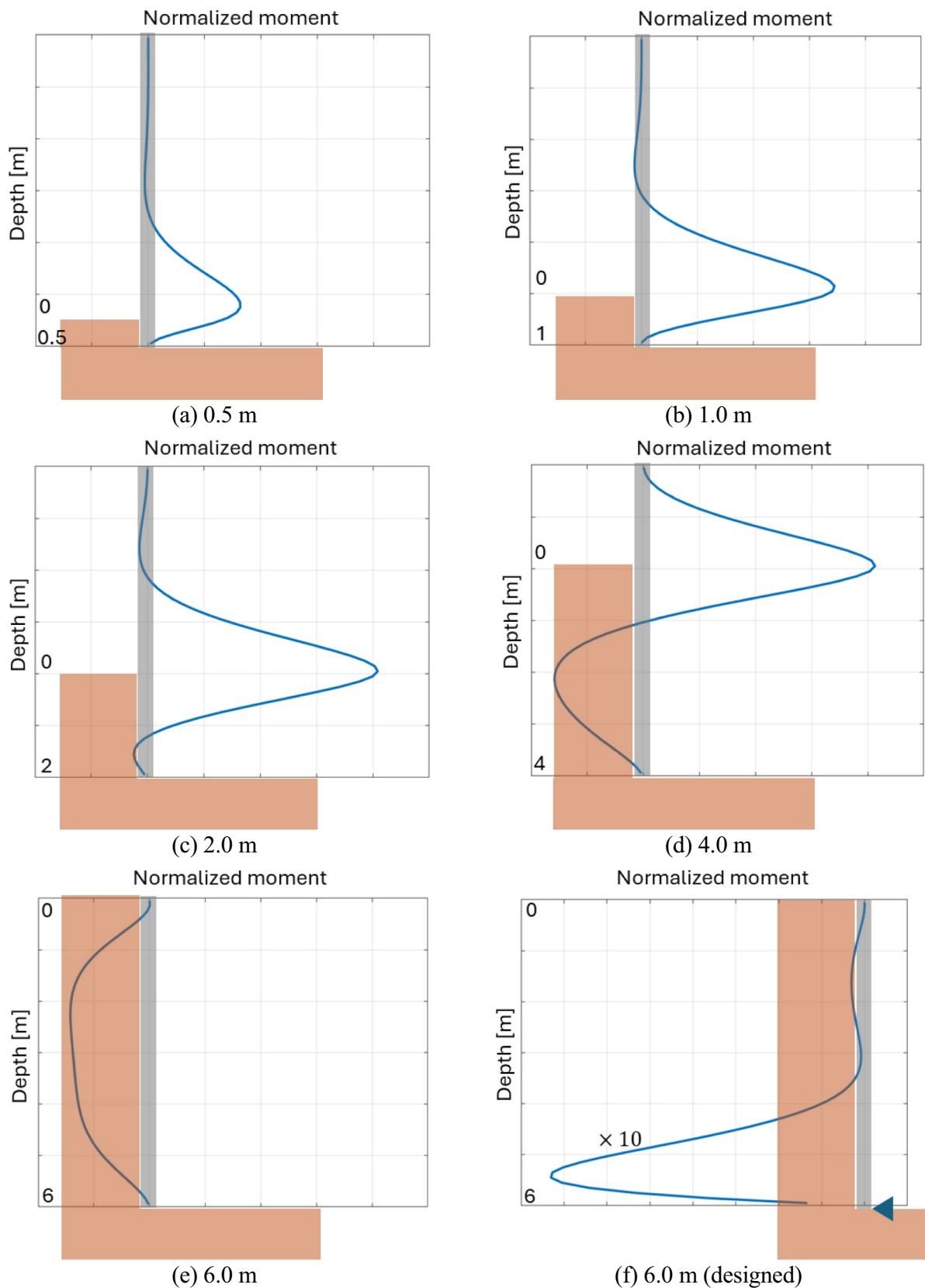


Fig. 3 Representative bending-moment distributions in the well wall during successive lowering stages: **a** embedment depth 0.5 m, **b** 1.0 m, **c** 2.0 m, **d** 4.0 m, **e** 6.0 m during lowering, and **f** the fully embedded post-plug reference state adopted for conventional structural design

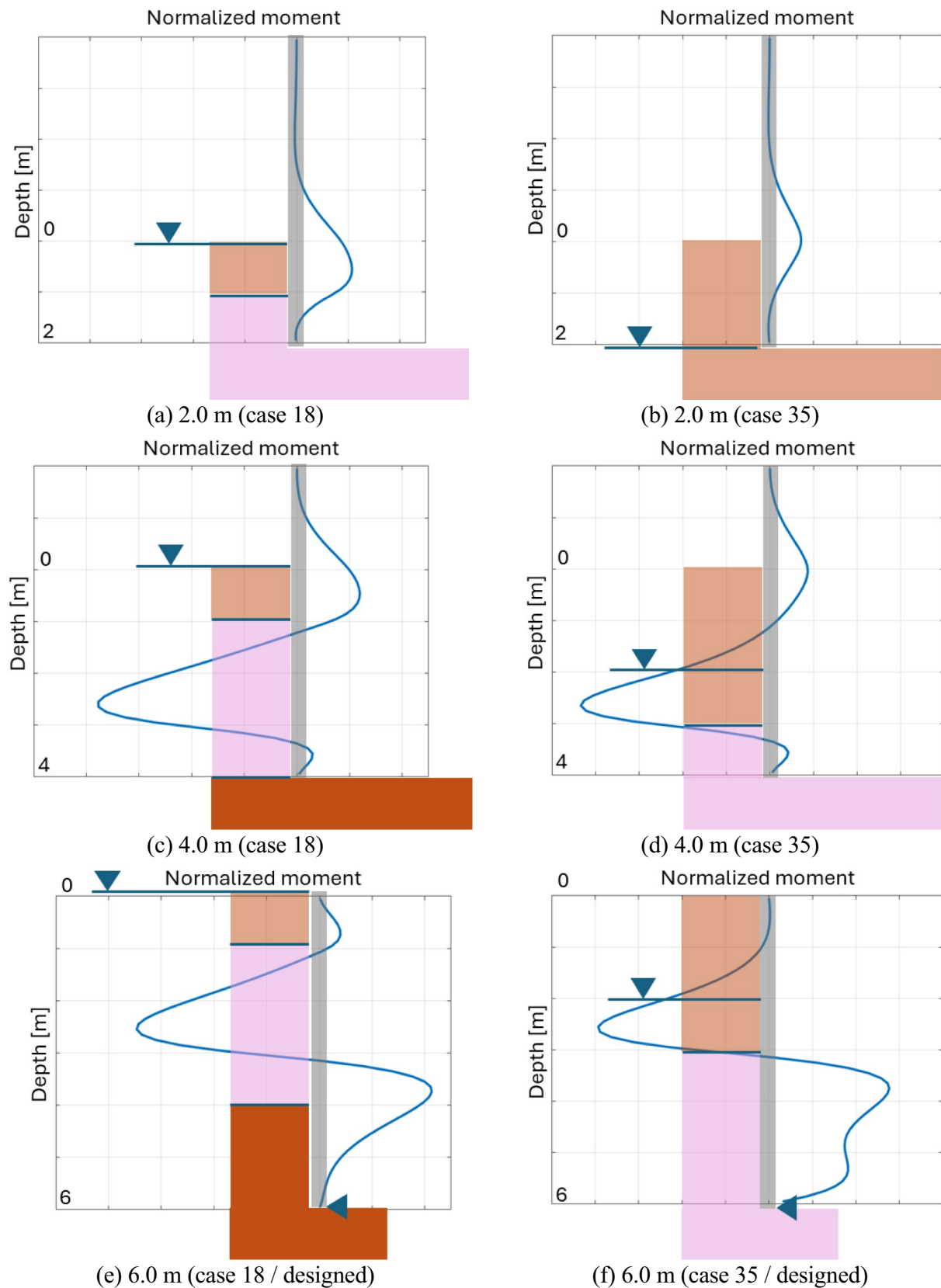


Fig. 4 Comparison of representative bending-moment distributions for two selected variants during lowering and in the final reference state: case 18 at embedment depths **a** 2.0 m, **c** 4.0 m, and **e** final post-plug state; case 35 at embedment depths, **b** 2.0 m, **d** 4.0 m, and **f** final post-plug state

rather than at full depth. In layered and groundwater-affected configurations, the peak moment exceeds the final design value, whereas in homogeneous dense sand conditions the final state remains conservative.

These observations motivated the introduction of $MAF(h)$, defined in Sect. 2.8, enabling systematic quantification across all 40 variants.

3.2 Global stage-dependent amplification

To quantify construction-stage effects relative to conventional design practice, the bending-moment amplification factor $MAF(h)$, defined in Sect. 2.8, was evaluated at every stage. The resulting amplification map for all variants and embedment depths is shown in Fig. 5. For clarity, Fig. 5 should be read row-wise, with each row representing one variant and each column corresponding to a successive embedment stage from 0.5 to 6.0 m. The figure is intended primarily as a global comparative map highlighting where amplification above unity appears, rather than as a detailed quantitative plot for any single case.

Figure 5 demonstrates that the evolution of bending moments is highly non-monotonic. In most configurations, internal forces increase during early embedment stages, reach a local maximum at intermediate depth, and subsequently decrease as the structural system approaches its final configuration.

The global maximum amplification observed across all variants is: $MAF_{max} = 1.16$ recorded for variants 21 and 31. This indicates that bending moments during lowering may exceed those obtained from the final post-plug configuration by up to 16%.

Importantly, amplification above unity is not uniformly distributed. Homogeneous dense sand configurations generally remain below $MAF = 1.0$, whereas layered soil profiles containing cohesive strata exhibit distinct amplification zones concentrated between 4.0 m and 5.5 m embedment depth.

This behavior confirms that the most unfavorable structural state may arise during transient construction stages rather than at final embedment.

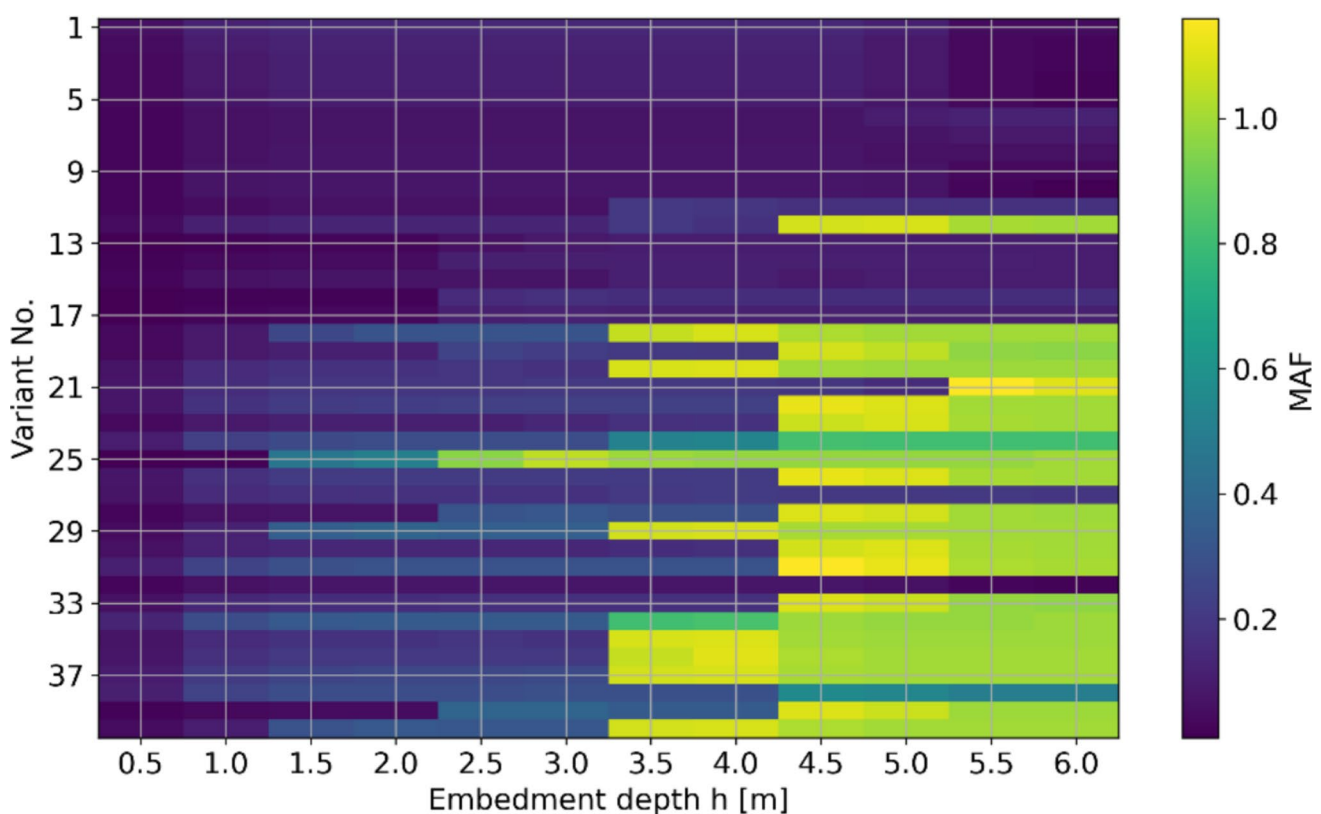


Fig. 5 Map of the stage-dependent bending-moment amplification factor $MAF = M(h)/M_{design}$ for all 40 geotechnical-hydrogeological variants and all embedment depths. Each row corresponds to one vari-

ant and each column to one embedment stage. Values greater than 1.0 identify stages for which the bending moment during lowering exceeds the conventional final post-plug design value

Table 3 Maximum MAF and corresponding critical embedment depth

Variant	Soil configuration	Water	MAF_{max}	h_{crit} [m]
21	S2/C1/S1	A	1.16	5.5
31	C1/S3	A	1.16	4.5
26	C1/S3/C2	D	1.12	4.5
22	S3/O/C2	D	1.12	4.5
36	S2/C2	C	1.11	4.0

3.3 Peak amplification and critical embedment depth

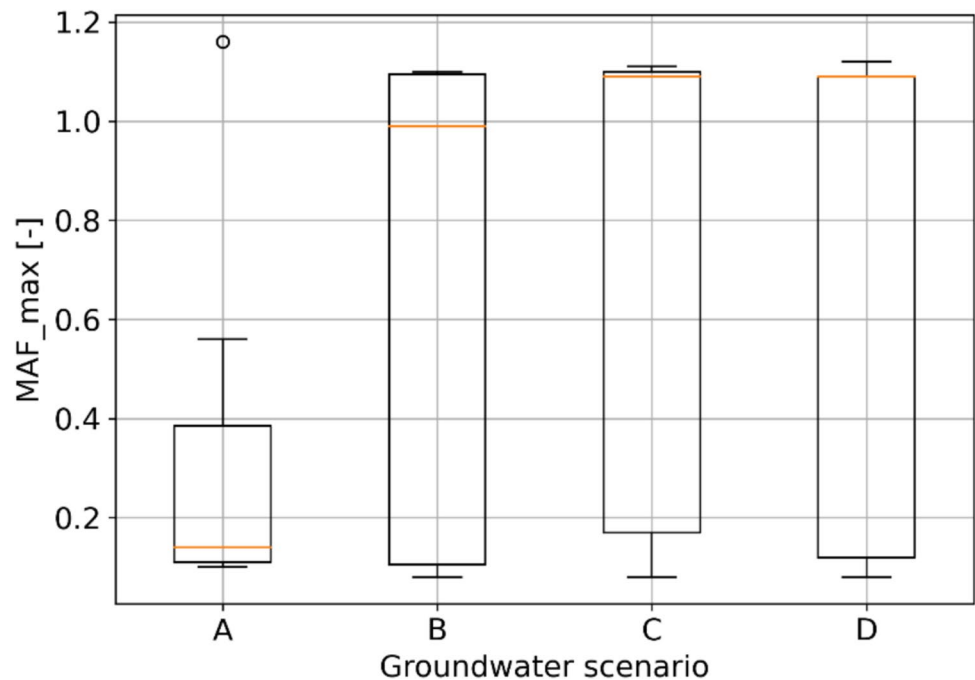
To synthesize the parametric study, the maximum MAF_{max} and corresponding critical embedment depth h_{crit} were extracted for each variant. Selected unfavorable cases are summarized in Table 3.

The results reveal that peak amplification is systematically associated with layered soil configurations involving stiffness contrast between adjacent strata. Even relatively thin cohesive layers (2–3 m) are sufficient to generate amplification above unity when located within the embedment range where radial pressure and friction are actively mobilized.

The critical embedment depth is consistently found between 4.0 and 5.5 m. At this stage:

- lateral earth pressure is substantially mobilized along the wall height,
- frictional resistance is active along a large contact area,

Fig. 6 Effect of groundwater scenario A–D on peak amplification MAF_{max}

**Table 4** Statistical summary of peak MAF by groundwater scenario

Scenario	Number of variants	Maximum MAF_{max}
A—Dry	11	1.16
B—Water at 4 m	11	1.10
C—Water at 2 m	9	1.11
D—Water at surface	9	1.12

- base reaction has not yet fully stabilized the structure,
- hydrostatic pressure may already contribute significantly.

This combination produces a temporary imbalance in radial load distribution, leading to increased bending demand.

3.4 Influence of groundwater level

The statistical influence of groundwater on peak amplification is illustrated in Fig. 6.

A clear upward shift in both median and upper-tail values is observed from dry conditions (A) toward saturated conditions (C and D). The dispersion of MAF values increases noticeably under elevated groundwater levels.

The maximum MAF values per groundwater scenario are summarized in Table 4.

Although the absolute maximum occurs under dry layered conditions (variant 21), elevated groundwater systematically increases the probability of amplification above unity. The mechanism is coupled:

1. Hydrostatic pressure increases radial loading on the wall.
2. Effective vertical stress is reduced, modifying sidewall friction.
3. Buoyant effects alter the balance between self-weight and uplift.

The combined effect intensifies bending demand in intermediate embedment stages, particularly in stratified soil systems.

3.5 Ranking of unfavorable configurations

The distribution of MAF_{max} across all 40 variants is shown in Fig. 7.

The ranking confirms that MAF above unity is restricted to specific soil–groundwater combinations. Approximately half of the analyzed variants remain below unity, indicating that final-state design is conservative in those cases.

The upper quartile of configurations is dominated by layered systems including soft plastic clay (C2) or organic soil (O), particularly when combined with groundwater scenarios C and D. This demonstrates that stiffness contrast rather than absolute soil strength governs MAF.

3.6 Structural interpretation and design implications

From a mechanical perspective, the observed MAF is governed by the evolving structural scheme of the well during lowering. In early stages, the wall behaves as a relatively short cylindrical shell with limited lateral confinement. As

embedment increases, radial earth pressure and friction mobilize along the shaft, while the base reaction is not yet fully effective.

This intermediate configuration resembles a free–free cylindrical shell subjected to nonuniform radial loading. After bottom plug construction, the boundary condition changes to a free–supported system, redistributing bending moments and generally reducing tensile stresses in the upper wall region. The identified MAF should be interpreted in view of the adopted loading idealization. Since no continuum soil domain is included, the model cannot reproduce arching effects or deformation-dependent redistribution of interface stresses. However, the observed non-monotonic evolution of bending moments still provides useful insight into the structural sensitivity of the well to changing external actions during lowering, especially when comparing homogeneous and stratified ground conditions within one consistent computational framework.

In several variants, bending sign reversal between lowering stages and final configuration was observed. Such behavior implies that reinforcement detailing based solely on final-state internal forces may not capture peak tensile demand during construction. The present results should also be interpreted with regard to the linear-elastic homogenized wall stiffness. In an actual reinforced concrete well, cracking would reduce local bending stiffness and may alter the redistribution of internal forces, especially in zones of repeated tensile demand. Thus, the reported amplification factors should be understood primarily as indicators of increased construction-stage demand relative to the

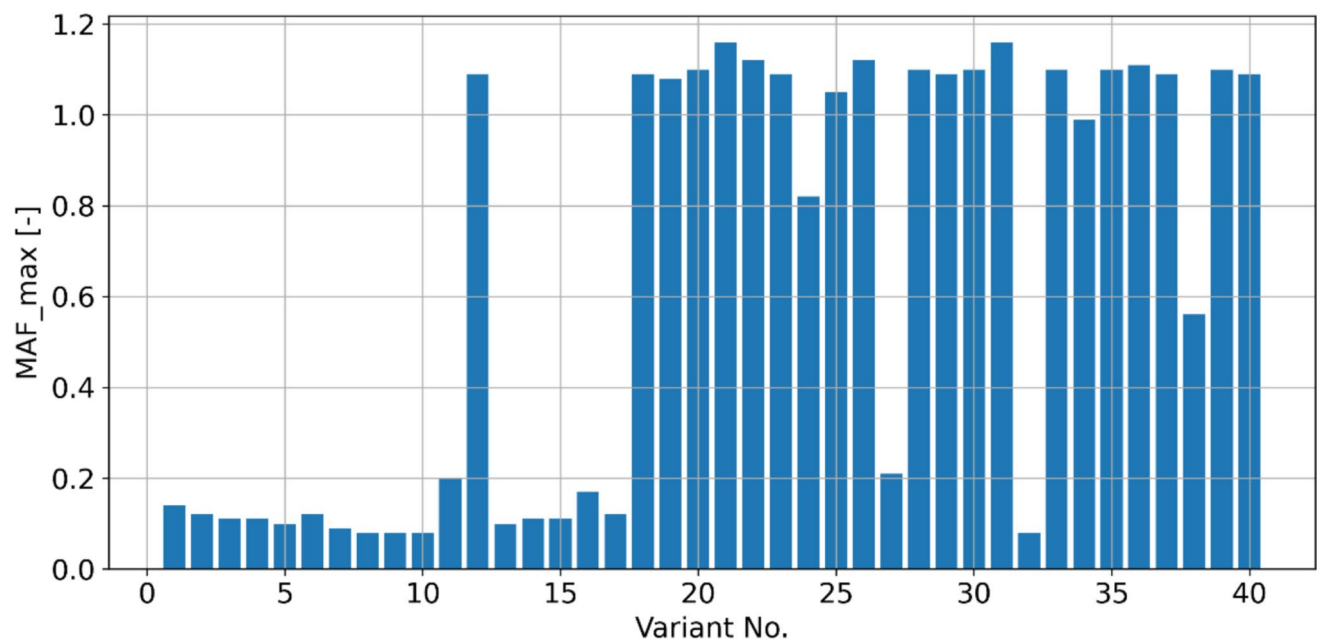


Fig. 7 Maximum bending moment amplification MAF_{max} for all variants

conventional final-state design, rather than as direct predictions of cracked-section response.

Overall, the parametric evidence demonstrates that:

- stage-dependent internal forces may exceed final design values by up to 16%,
- amplification is governed primarily by stratification-induced stiffness contrast,
- groundwater acts as a secondary but systematic amplification mechanism,
- and the critical structural state occurs during intermediate embedment rather than at full depth.

These findings support the inclusion of staged numerical assessment in the design of sunk wells constructed in layered and groundwater-affected soils. It should be emphasized that the present formulation does not include a deformable soil continuum and therefore cannot reproduce all aspects of soil-structure interaction observed in the field. In particular, local soil deformation patterns, stiffness-dependent redistribution in the surrounding ground, and feedback between wall displacement and contact stress evolution are not resolved directly. Nevertheless, the adopted approach remains useful for identifying relative amplification trends and for highlighting the construction stages at which the structural response becomes most unfavorable under prescribed loading scenarios.

4 Conclusions

This study presented a parametric axisymmetric finite element investigation of reinforced concrete sunk wells subjected to stage-dependent soil and groundwater actions during the lowering process. Unlike conventional design approaches that focus exclusively on the final post-plug configuration, the proposed framework quantified the evolution of internal forces throughout incremental embedment.

On the basis of 40 geotechnical–hydrogeological variants, the following conclusions can be drawn:

1. The maximum bending moment amplification factor reached $MAF_{max} = 1.16$ indicating that bending moments during lowering may exceed conventional post-plug design values by up to 16%. The critical demand was consistently observed at intermediate embedment depths (typically 4.0–5.5 m), rather than at full depth.

2. Homogeneous dense sand profiles produced stable, nearly monotonic moment evolution with $MAF_{max} < 1.0$. In contrast, layered systems involving soft plastic clay interlayers generated localized MAF and sign reversal. The decisive parameter is the stiffness contrast between layers, which induces nonuniform radial load redistribution along the shaft.
3. Elevated groundwater levels increase hydrostatic pressure while simultaneously reducing effective stress and frictional resistance. This dual effect results in both higher peak bending moments and greater dispersion between variants. Saturated layered configurations represent the most unfavorable scenario.
4. During lowering, the well behaves as a free–free cylindrical shell, whereas after plug construction it transforms into a free–supported system. The change in static scheme alters both the magnitude and location of tensile stresses. In several variants, bending sign reversal was observed between lowering and final states, which has direct implications for reinforcement detailing.
5. Reinforcement design based solely on the final configuration may underestimate tensile demand in the upper and middle wall zones during intermediate construction stages, particularly in stratified saturated soils. Incorporating stage-dependent assessment into the design process improves reliability without requiring full soil-continuum modeling.

In summary, the results demonstrate that the lowering phase represents a structurally critical transient state that cannot be inferred from the final equilibrium configuration alone. The proposed axisymmetric staged framework provides a computationally efficient tool for identifying unfavorable soil–groundwater combinations and quantifying their impact on bending demand.

Further refinement of predictive capability would require extension toward nonlinear material behavior, geometric imperfections, and fuller soil-structure interaction representation, which remain outside the scope of the present comparative study.

Appendix

See Table 5.

Table 5 Percentage values of the bending moment ratios

No.	0.5 m (%)	1.0 m (%)	1.5 m (%)	2.0 m (%)	2.5 m (%)	3.0 m (%)	3.5 m (%)	4.0 m (%)	4.5 m (%)	5.0 m (%)	5.5 m (%)	6.0 m (%)
1	6	12	14	14	14	14	14	14	14	11	5	5
2	5	10	12	12	12	12	12	12	12	9	4	3
3	4	9	11	11	11	11	11	11	11	9	4	3
4	4	9	11	11	11	11	11	11	11	9	4	2
5	4	8	10	10	10	10	10	10	10	8	4	2
6	3	6	7	7	7	7	7	7	7	10	12	12
7	3	6	7	7	7	7	7	7	7	7	9	9
8	3	6	8	8	8	8	8	8	8	6	6	6
9	3	7	8	8	8	8	8	8	8	7	4	4
10	3	7	8	8	8	8	8	8	8	7	3	1
11	3	5	6	6	6	6	20	19	17	17	18	18
12	5	11	13	13	13	13	20	18	108	109	101	100
13	1	2	3	3	6	9	10	10	10	10	10	10
14	2	4	5	5	11	11	11	11	11	11	11	10
15	3	6	7	7	7	7	11	11	9	10	10	10
16	1	2	2	2	15	17	16	16	16	16	16	16
17	1	3	4	4	11	12	11	11	11	11	11	11
18	4	9	26	31	31	31	106	109	102	100	100	100
19	4	9	11	11	24	22	20	20	108	105	97	96
20	7	15	18	18	19	18	109	110	100	99	99	99
21	8	16	19	19	19	20	20	20	19	15	116	111
22	8	18	21	22	23	23	23	23	112	110	100	100
23	4	9	11	11	11	14	17	18	107	109	101	100
24	10	23	27	28	28	28	52	53	82	81	81	81
25	1	2	46	51	96	105	99	98	98	98	98	100
26	8	17	21	21	21	21	21	21	112	110	101	100
27	7	15	17	18	18	18	20	21	20	20	20	19
28	3	6	7	7	31	32	29	29	110	108	100	99
29	5	12	36	38	36	36	108	109	101	101	100	100
30	6	12	14	14	14	14	16	17	108	110	101	100
31	11	24	29	30	30	30	30	30	116	112	102	101
32	3	6	8	8	8	8	8	8	8	6	3	2
33	6	14	16	16	17	17	17	17	110	107	98	97
34	13	28	33	34	34	34	81	83	99	98	98	99
35	7	15	18	18	19	18	109	110	100	99	99	99
36	8	17	20	20	20	20	106	111	102	100	100	100
37	10	21	25	26	26	27	108	109	101	100	100	100
38	11	24	28	28	28	29	29	29	56	54	50	50

Table 5 (continued)

No.	0.5 m (%)	1.0 m (%)	1.5 m (%)	2.0 m (%)	2.5 m (%)	3.0 m (%)	3.5 m (%)	4.0 m (%)	4.5 m (%)	5.0 m (%)	5.5 m (%)	6.0 m (%)
39	2	4	5	5	38	38	34	34	110	107	99	99
40	5	10	30	33	32	32	108	109	102	<i>100</i>	<i>100</i>	<i>100</i>

Bold indicates scenarios where the stresses induced during the sinking of the well exceed the reference values assumed in traditional design

Italic indicates scenarios in which the stresses are close to the reference values, i.e., within 95% to 100% of the reference values.

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Data availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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