STOCHASTIC MODEL REDUCTION TECHNIQUES APPLIED TO INVERSE PROBLEMS IN STRUCTURAL ENGINEERING

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1. Introduction

In the present paper the stochastic programming technique based on Gaussian Processes [1, 2] applied to inverse problems in structural engineering, e.g. material parameters characterization and damage detection is presented. The inverse analysis often uses a numerical model as an counterpart of experiment in order to build the discrepancy function between experimentally measured and numerically computed quantities, such as displacements, reaction forces, strains, accelerations, etc. If the numerical model is complex the iterative minimization procedure becomes very expensive, therefore not attractive from practical point of view or when the test has to be performed 'in situ' (i.e. without a computer which can handle heavy computations). The alternative is to use a surrogate which approximates the behavior of the numerical model but is much simpler thus less expensive. The surrogate is usually constructed as a 'black box' where for the approximation the following methods, among others, are commonly used: Radial Basis Functions (RBFs), Polynomials, Proper Orthogonal Decomposition (POD) combined with RBFs, Artificial Neural Networks (ANNs) or Gaussian Processes (GP).

All listed here approximation techniques require the numerically computed responses (i.e. training samples) in order to build a smooth and accurate analytical approximation of the sought solution. Ideally would be to use a method which need the smallest possible number of 'training' points and in the same time is precise and robust. The approximation method based on GP satisfies all above mentioned requirements: it gives very good results when the number of training examples is limited. Another important feature of GP is that it gives not only the approximation of the mean value of sought parameter but also its standard deviation. This feature gives a possibility of automatic and systematic improvement of the solution, because the computed standard deviation of the model prediction provides a localization where the approximation is weak, (and therefore it points out where, in the parameter space, the additional experimental or numerical data are necessary to improve the approximation).

An important problem during the construction of the surrogate is usually a big number of data, i.e. control parameters (e.g. material, geometrical features) and state parameters (measurable quantities). The probable correlations between the control variables as well as between the state variables can be computed, and consequently used to reduce the number of model parameters, by the application of Principal Component Analysis (which is a part of the proposed method). The presented stochastic algorithm is formulated within Bayesian framework thus provides additional information about the magnitude of correlation between state and control variables, i.e. the relevance of input-output correlation. This is very important if one would like to exclude from the model the parameters which not influence the measurable quantities (i.e. the measurable quantities are not sensitive to those parameters).

The stochastic model reduction techniques based on GP have, however, one significant disadvantage, namely the Gaussian Processes are usually parameterized in terms of their covariance functions. This makes it difficult to deal with multiple outputs, because ensuring that the covariance matrix is positive definite is problematic. An alternative formulation is to treat Gaussian processes as white noise sources convolved with smoothing kernels, and to parameterize the kernel instead (see [3]).

Using this approach, one can extend Gaussian Processes to handle multiple, coupled outputs.

2. Application

In the present communication two examples are used to show the application of above described model reduction techniques. The first example shows the application of multi-output GP to damage detection in the structural elements (as beams and plates) through Wavelet Transformation [4, 5] and Inverse Analysis. The second application shows the use of GP as numerical model surrogate in characterization of glass and foil parameters in SGP and PVP laminated glasses [6, 7] through Digital Image Correlation and Inverse Analysis [8].

In both examples GP based approximation serves as a surrogate of numerical model, which in combination with iterative minimization algorithm (e.g. trust-region algorithm) gives very fast and accurate results, both in damage detection and material model parameters identification. By iterative comparing of experimental data to data obtained from the multi-output GP approximation model the discrepancy is minimized and sought parameters (i.e. damage localization and size, as well as material constants in laminated glass) can be vary fast identified, provided the surrogate is appropriate constructed.

3. Summary

The GP approximation model which serve as a numerical model reduction is used here in combination with Inverse Analysis to solve structural engineering problems, e.g. damage detection and constitutive models identification. The work is mainly focus on the proper construction of the GP model, namely on: (1) training process based on minimal number of training samples, by making use of automatic samples selection through computed standard deviation of model prediction; (2) control and state parameters compression based on PCA techniques; (3) control parameters reduction based on input-output correlation; (4) proper construction of multi-output GP.

4. References

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