

Article

Directional Anisotropy of Admissible Tilt in Rectangular Sinking Wells: A Closed-Form Analytical Model

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Abstract

Rectangular sinking wells are widely used in underground and hydraulic engineering, where maintaining vertical alignment during construction is essential for safe serviceability and long-term performance. Even moderate inclination may cause eccentric transfer of the vertical load to the concrete plug, leading to non-uniform contact stresses beneath the base. This study presents a closed-form analytical framework for assessing the admissible tilt of rectangular sinking wells based on stress redistribution under one-axis inclination. The well is modeled as a rigid body, and the resulting load eccentricity is related to the shaft height and tilt angle. Classical eccentric compression theory is then used to derive explicit expressions for the maximum and minimum base stresses, from which a serviceability-based admissibility criterion is formulated. The obtained solution provides the allowable inclination directly as a function of shaft height, plan dimensions, and an adopted stress amplification factor. Parametric analyses show that the admissible tilt decreases with increasing shaft height and increases with larger base dimensions. A distinct directional anisotropy is observed for rectangular plans, whereas square wells recover symmetric behavior with identical limits in both orthogonal directions. This identifies square geometry as the symmetry-preserving limit case of the proposed model. Sensitivity analyses further demonstrate the influence of admissible stress amplification on permissible inclination levels. The proposed formulation offers a transparent screening tool for construction monitoring and post-construction assessment, while also illustrating how geometric symmetry reduction governs the mechanical response of inclined foundation systems.

Keywords: rectangular sinking wells; inclination assessment; base stress redistribution; eccentric loading; admissible tilt; underground structures



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1. Introduction

The rapid development of modern civil engineering and geotechnics is closely associated with the need to safely transfer increasingly large structural loads to complex and often heterogeneous soil strata [1,2]. In contemporary urban environments, characterized by high building density and the growing demand for underground infrastructure, traditional open excavations are frequently technically difficult or economically impractical [3,4]. These challenges become particularly significant in areas with high groundwater levels or in coastal regions, where the control of water inflow and the stabilization of excavation walls

may pose considerable risks to surrounding structures and existing infrastructure [5,6]. Under such conditions, reinforced concrete open caissons, referred to as sinking wells, represent a reliable and widely used structural solution [4,6,7].

The technology of sinking wells has evolved from early brick-lined shafts to modern reinforced concrete caisson structures designed as monolithic or segmented systems [6,8]. Today they are commonly used as structural enclosures for sewage pumping stations, technological chambers, bridge piers, and ventilation shafts in underground transport systems [9–11]. The principal advantage of this construction method lies in the so-called “cut and lower” technique, which enables the execution of deep underground structures while limiting disturbances to the surrounding soil mass and reducing the need for extensive temporary excavation support systems [12,13]. Nevertheless, the sinking process constitutes a complex geomechanical operation that requires careful control of technological parameters and a detailed understanding of soil structure interaction mechanisms [14,15].

During the sinking phase, the structural behavior of the well differs significantly from its final operational state. At this stage the structure acts essentially as a thin-walled reinforced concrete shell [16,17]. The walls of the well are subjected to a combination of lateral earth pressure, hydrostatic pressure, and skin friction acting along the external surface of the structure [18,19]. The magnitude of skin friction is particularly difficult to predict because it depends on several factors, including soil type, moisture conditions, and the roughness of the well surface [20,21]. In practice, controlled descent of the structure is achieved by balancing the structural weight against these resisting forces, often supported by the application of specialist agents such as thixotropic bentonite suspensions that reduce friction at the soil structure interface [22,23].

A fundamental change in the structural behavior occurs once the well reaches its design founding level. The construction of the concrete bottom plug transforms the structural system from a vertically movable shell into a rigid foundation block [19,24]. At this stage the entire vertical load including the self-weight of the shaft, the mass of the bottom plug, installed technological equipment, and possible superstructure loads, is transmitted to the soil through direct contact at the base of the structure [25–27]. Consequently, the interaction between the base slab and the supporting soil becomes the primary factor governing settlement behavior, stress distribution, and long-term structural performance [28,29].

In modern hydraulic and environmental engineering practice, the geometry of sinking wells is often determined by technological requirements associated with the equipment located inside the structure [30,31]. Although circular wells are traditionally preferred due to their axisymmetric geometry, which facilitates uniform stress distribution and reduces structural complexity, rectangular sinking wells are increasingly employed in engineering practice [6,32]. Their growing popularity is mainly associated with more efficient use of internal space and easier installation of pumping equipment, pipelines, and electrical systems [33,34]. However, the rectangular geometry introduces additional structural challenges [27,35]. Due to the lack of axial symmetry, the contact stresses at the base slab are inherently non-uniform, and significant stress concentrations may occur, particularly in corner regions [28,36]. These stress concentrations are highly sensitive to deviations from vertical alignment, making the control and assessment of verticality an important aspect of both design and construction monitoring [37,38].

Maintaining perfectly vertical sinking conditions in field practice is difficult. Geological variability, such as the occurrence of boulders, stiff clay lenses, or heterogeneous sand layers, may cause asymmetric resistance at the cutting-edge during excavation [7,32]. Additionally, unavoidable irregularities in the soil removal process inside the well may result in small but significant deviations from the vertical axis [17]. Any inclination of a rectangular well leads to an eccentric transfer of vertical loads to the base slab [19,38]. As

a result, the contact pressure distribution beneath the foundation becomes non-uniform and approximately linear across the base. If the inclination exceeds certain limits, stress amplification at the leading edge of the foundation may exceed the bearing capacity of the soil, potentially causing local failure or excessive differential settlement [25,31]. At the same time, the opposite edge of the foundation may experience a reduction or even complete loss of contact with the soil, leading to the so-called lift-off phenomenon, which significantly alters the stiffness of the soil structure system [34,36].

Although advanced three-dimensional numerical simulations based on the Finite Element Method (FEM) and nonlinear soil models can capture these phenomena in detail, their practical use during construction monitoring is often limited by computational complexity and the need for specialized expertise [35,36]. Although the underlying mechanics of eccentric loading is well established in classical foundation theory, its direct application to the rapid assessment of inclined rectangular sinking wells has not been sufficiently formalized in a compact engineering form. In particular, there is still a practical need for transparent relationships that allow engineers to estimate the consequences of measured inclination in terms of base stress amplification and admissible tilt limits [6,31,38]. Existing studies and design recommendations typically focus on the internal forces in well walls or on the settlement of perfectly vertical foundations, while intermediate states associated with structural tilt receive relatively limited attention [19,27,33].

It should be emphasized that the present study does not aim to replace classical eccentric loading theory or to introduce a new contact mechanics model. Instead, its contribution lies in reformulating these well-established relationships into a dedicated closed-form assessment procedure for inclined rectangular sinking wells. In this sense, the novelty of the work is threefold: first, it directly links the measured inclination angle of the shaft with the stress amplification beneath the concrete plug; second, it expresses admissible inclination in terms of a simple serviceability-oriented stress amplification criterion; and third, it highlights the directional anisotropy of rectangular wells, for which the admissible tilt depends on the plan dimension in the considered direction. The proposed formulation is therefore intended as a transparent screening tool for engineering assessment, rather than as a replacement for detailed numerical soil–structure interaction analysis.

Modern design standards, including Eurocode 7, impose strict requirements regarding both ultimate limit states (ULS) and serviceability limit states (SLS) for deep foundation systems [21,26,37]. For rectangular wells, serviceability criteria are often decisive because uneven settlements caused by inclination may lead to leakage in pipeline connections, malfunction of installed technological equipment, or damage to monitoring systems located within the structure [30,31].

Recent studies have increasingly focused on the behavior and installation mechanisms of caisson foundations under complex loading and ground conditions. Li et al. [39] and Zhang et al. [30] investigated the sinking mechanisms and control of large-scale caissons in sandy soils, emphasizing the role of soil–structure interaction and construction-induced effects. Experimental work by Swallow et al. [40] provided insight into the development of skin friction during caisson sinking in sands. In the offshore context, Cheng et al. [19] analyzed suction caisson anchors under inclined loading in calcareous sands, highlighting the influence of particle breakage on load capacity. Furthermore, Zhou et al. [12] proposed an analytical solution for soil deformation induced by pipe jacking, accounting for the effect of grouting pressure. Despite these advances, the admissible inclination of rectangular sinking wells and its influence on base stress redistribution remain insufficiently addressed particularly in the context of simple analytical approaches suitable for practical engineering assessment.

From a mechanics perspective, rectangular wells constitute a particularly interesting case of symmetry reduction. Unlike circular or square systems, the unequal plan dimensions generate direction-dependent stress amplification under identical tilt angles. This motivates the present analytical study not only from an engineering viewpoint, but also as a compact example of geometry-controlled anisotropic response.

In response to these challenges, this study proposes a compact analytical framework for assessing the admissible inclination of rectangular sinking wells. By interpreting the tilt of the structure as an eccentric transfer of vertical load, a closed-form analytical expression describing the resulting amplification of contact stresses at the foundation base is derived. The proposed admissibility criterion relates the stress increase caused by inclination to the reference stress state corresponding to a perfectly vertical configuration, providing engineers with a practical tool for rapid assessment during construction monitoring as well as for post-construction evaluation of structural safety [35,38].

2. Materials and Methods

2.1. General Assumptions and Geometric Description

The analyzed structure is a reinforced concrete sinking well with a rectangular plan geometry, characterized by the dimensions: B —width of the well, L —length of the well, H —shaft height, t —wall thickness, h_c —thickness of the concrete bottom plug.

The well is assumed to have reached its final foundation depth and to be completed with a rigid concrete plug at the bottom. The plug is assumed to be fully bonded to the shaft, and its upper surface is assumed horizontal.

The total permanent vertical load transmitted to the soil consists of: G_s —self-weight of the shaft, G_p —weight of the cover slab and additional permanent surcharge or superstructure load, G_c —self-weight of the concrete plug.

Thus, the total vertical force acting on the soil through the plug is

$$N = G_s + G_p + G_c. \quad (1)$$

The plan area of the rectangular plug is

$$A = BL \quad (2)$$

For the perfectly vertical well, the load is transferred concentrically and the contact stress beneath the plug is assumed uniform:

$$\sigma_0 = \frac{N}{A}. \quad (3)$$

The reference stress σ_0 corresponds to uniform contact pressure under concentric loading.

2.2. Evaluation of Component Weights

In order to retain the physical interpretation of the model, the individual load components are expressed in terms of geometry and material parameters.

The shaft is modeled as a hollow rectangular prism of outer dimensions $B \times L$ and wall thickness t . Its inner dimensions are therefore

$$B_i = B - 2t, \quad (4)$$

$$L_i = L - 2t. \quad (5)$$

The shaft volume is

$$V_s = (BL - B_i L_i)H, \quad (6)$$

and its self-weight equals

$$G_s = \gamma_c V_s, \quad (7)$$

where γ_c is the concrete unit weight.

The plug volume is

$$V_c = BLh_c, \quad (8)$$

and the corresponding self-weight is

$$G_c = \gamma_c V_c. \quad (9)$$

The permanent load associated with the cover slab, superstructure or surcharge is represented in equivalent form by a uniform surface load q_p , giving

$$G_p = q_p BL. \quad (10)$$

Substituting Equations (7), (9) and (10) into Equation (1) yields the total load transmitted to the soil. The above expressions define only the magnitude of the vertical load components. In the basic formulation, these loads are assumed to be symmetrically distributed with respect to the centroid of the plug. In practice, additional eccentricity may also arise independently of shaft inclination, for example due to asymmetric equipment layout, non-uniform surcharge, construction tolerances, or local stiffness variations. Such effects are not included in the reference formulation, which isolates the eccentricity caused by inclination. If required, an additional load-induced eccentricity may be superimposed on the inclination-induced eccentricity in a project-specific assessment.

2.3. Single-Axis Inclination and Equivalent Eccentricity

The present study considers single-axis inclination, which means that at a given time the well is analyzed as tilted in one vertical plane only.

To preserve a compact formulation applicable to both possible directions, a generic plan dimension d is introduced, where:

$$d = L \quad (\text{for tilt in the longitudinal direction}), \quad (11)$$

$$d = B \quad (\text{for tilt in the transverse direction}). \quad (12)$$

Let α denote the tilt angle in the considered plane. Assuming rigid-body rotation of the shaft and small tilt angles, the corresponding eccentricity of the resultant vertical load with respect to the centroid of the plug is written as

$$e = \frac{H}{2} \tan \alpha. \quad (13)$$

For small angles

$$\tan \alpha \approx \alpha, \quad (14)$$

when α is expressed in radians. This approximation is valid for small angles typically below 5° , which corresponds to practical engineering cases. The expression (13) assumes that the resultant vertical load acts at mid-height of the shaft, which is consistent with uniform mass distribution and typical engineering approximation.

This assumption should be interpreted as an equivalent resultant approximation. It is most appropriate when the self-weight of the shaft dominates the vertical load distribution and the mass is approximately uniform along the height. If substantial localized loads, heavy equipment, or non-uniform superstructure actions are present, the effec-

tive point of application of the vertical resultant may differ from $H/2$. In such cases, Equation (13) may be used with an effective lever arm z_N instead of $H/2$, provided that this value is determined from the actual vertical load distribution. In the present study, the mid-height assumption is retained to preserve the closed-form character of the proposed screening model.

The bending moment induced at the base is

$$M = Ne. \quad (15)$$

2.4. Section Properties for One-Direction Bending

For a rectangular base subjected to one-direction eccentric loading, the stress varies linearly along the direction of tilt.

To preserve a unified notation, the dimension perpendicular to the direction of stress variation is denoted by b , and the dimension along which the stress varies is denoted by d . Thus for tilt along L : $b = B$, $d = L$, and for tilt along B : $b = L$, $d = B$.

The second moment of area with respect to the relevant centroidal axis is

$$I = \frac{bd^3}{12}, \quad (16)$$

and the corresponding section modulus is

$$W = \frac{I}{d/2} = \frac{bd^2}{6}. \quad (17)$$

Hence, for the two particular tilt directions:

Tilt along L

$$W_L = \frac{BL^2}{6}, \quad (18)$$

tilt along B

$$W_B = \frac{LB^2}{6}. \quad (19)$$

2.5. Contact Stress Distribution Under One-Direction Eccentric Loading

This assumption corresponds to the classical rigid footing theory under elastic-perfectly compressive soil response and is commonly used in engineering practice for preliminary assessment. The linear stress distribution should be understood as a rigid-footing approximation. It is most representative for preliminary serviceability-level assessment when the plug and shaft may be treated as significantly stiffer than the supporting soil response and when full compressive contact is maintained beneath the base. In Winkler-type or elastic-continuum foundation models, the same tendency of stress increase on the loaded side and stress reduction on the opposite side is obtained, although the detailed stress field may depend on soil stiffness distribution, boundary conditions, and foundation flexibility. For very soft soils, strongly nonlinear settlement response, or partial loss of contact, the present linear formulation should therefore be treated only as a first screening estimate.

The stress distribution beneath the base for one-direction eccentric loading is assumed linear and is expressed by the classical relationship

$$\sigma = \frac{N}{A} \pm \frac{M}{W}. \quad (20)$$

Substituting Equations (2), (15) and (17) gives

$$\sigma = \frac{N}{bd} \pm \frac{6Ne}{bd^2}. \quad (21)$$

Thus, the maximum and minimum contact stresses are

$$\sigma_{max} = \frac{N}{bd} + \frac{6Ne}{bd^2}, \quad (22)$$

$$\sigma_{min} = \frac{N}{bd} - \frac{6Ne}{bd^2}. \quad (23)$$

Dividing by the reference stress $\sigma_0 = N/(bd)$ yields the normalized form

$$\frac{\sigma_{max}}{\sigma_0} = 1 + \frac{6e}{d}, \quad (24)$$

$$\frac{\sigma_{min}}{\sigma_0} = 1 - \frac{6e}{d}. \quad (25)$$

These equations show that the stress amplification depends only on the ratio e/d , and therefore on the tilt direction and the corresponding plan dimension.

2.6. Serviceability Admissibility Criterion

To evaluate whether the inclined well may remain in service, a stress-based admissibility criterion is introduced by limiting the maximum contact stress beneath the plug:

$$\sigma_{max} \leq k_\sigma \sigma_0, \quad (26)$$

where k_σ is the allowable stress amplification factor. The factor k_σ represents the admissible amplification of maximum base stress relative to the concentric reference state. Its value depends on project requirements, soil deformability, and acceptable settlement risk. In this study,

$$k_\sigma = 1.2, \quad (27)$$

is adopted as a conservative benchmark value corresponding to a moderate 20% stress increase. In this study, $k_\sigma = 1.2$ is adopted as a conservative benchmark corresponding to a moderate 20% increase in the maximum base stress relative to the concentric reference state. This value should not be interpreted as a direct code-prescribed limit. Rather, it represents a serviceability-oriented engineering threshold suitable for preliminary assessment, consistent with the general design philosophy in which excessive stress redistribution and differential settlement should be limited. In practical applications, the value of k_σ should be selected by the designer with regard to soil conditions, monitoring accuracy, acceptable settlement risk, and project-specific serviceability requirements. To illustrate the influence of this parameter, the analytical results are additionally discussed for a broader range of k_σ values. To demonstrate robustness, a sensitivity study for $k_\sigma = 1.1$ – 1.5 is additionally presented.

Substituting Equation (24) into Equation (26) gives

$$1 + \frac{6e}{d} \leq k_\sigma, \quad (28)$$

and therefore

$$e \leq \frac{(k_\sigma - 1)d}{6}. \quad (29)$$

Equation (29) defines the admissible eccentricity for the considered tilt direction. It should be noted that the proposed criterion does not directly address ultimate bearing capacity of the soil, but rather provides a serviceability-based limit relative to the reference stress state.

2.7. Admissible Tilt Angle

Using Equation (13), the eccentricity criterion may be transformed into the angle space:

$$\frac{H}{2} \tan \alpha \leq \frac{(k_\sigma - 1)d}{6}, \quad (30)$$

which gives

$$\tan \alpha \leq \frac{(k_\sigma - 1)d}{3H}. \quad (31)$$

Hence, the admissible tilt angle is

$$\alpha_{\text{lim}} = \arctan\left(\frac{(k_\sigma - 1)d}{3H}\right). \quad (32)$$

This equation provides the basic closed-form engineering tool of the present study. For the two orthogonal directions, the projected limits become:

Tilt along L

$$\alpha_{\text{lim},L} = \arctan\left(\frac{(k_\sigma - 1)L}{3H}\right), \quad (33)$$

tilt along B

$$\alpha_{\text{lim},B} = \arctan\left(\frac{(k_\sigma - 1)B}{3H}\right). \quad (34)$$

Equations (33) and (34) show directly that rectangular wells exhibit a direction-dependent admissible inclination.

2.8. Full-Contact Condition and Kern Limit

In addition to the serviceability condition, it is useful to verify whether the contact beneath the plug remains fully compressive. In the present one-direction formulation, full contact is preserved as long as

$$\sigma_{\text{min}} \geq 0. \quad (35)$$

Using Equation (25), this gives

$$1 - \frac{6e}{d} \geq 0, \quad (36)$$

or

$$e \leq \frac{d}{6}. \quad (37)$$

Equation (37) corresponds to the classical kern limit of the rectangular base. Comparison of Equation (29) with Equation (37) shows that the serviceability-based admissible eccentricity remains well below the geometric limit associated with loss of contact. For the reference case $k_\sigma = 1.2$,

$$e_{\text{lim}} = \frac{d}{30}, \quad (38)$$

which represents only 20% of the kern limit.

The corresponding normalized minimum stress at the admissible state is obtained from Equation (25):

$$\frac{\sigma_{min}}{\sigma_0} = 2 - k_\sigma, \quad (39)$$

and for $k_\sigma = 1.2$,

$$\frac{\sigma_{min}}{\sigma_0} = 0.8. \quad (40)$$

Thus, even at the admissible tilt limit, the minimum stress remains positive and significant, confirming that the adopted criterion is conservative with respect to loss of contact.

2.9. Analytical Expression for Directional Admissible Tilt

Because the admissible tilt depends on the plan dimension in the considered direction, the rectangular well exhibits directional anisotropy.

A convenient scalar measure of this effect is the ratio of the one-axis admissible projections:

$$\eta_\alpha = \frac{\alpha_{lim,L}}{\alpha_{lim,B}}. \quad (41)$$

For small angles, using Equations (33) and (34), one obtains approximately

$$\eta_\alpha \approx \frac{L}{B}. \quad (42)$$

This result shows that the geometry of the plan alone governs the directional sensitivity of the one-axis tilt response. Square wells are therefore expected to exhibit nearly symmetric behavior, whereas elongated wells should have markedly different admissible inclinations in the two principal directions.

2.10. Scope and Role of the Proposed Model

The analytical framework presented above is based on the following assumptions:

1. the well behaves as a rigid body,
2. the plug is rigid and fully bonded to the shaft,
3. the base stress distribution is linear in the tilt direction,
4. only one tilt component is considered at a time,
5. the soil response beneath the plug remains compressive within the admissible range,

Within these assumptions, the model provides a transparent closed-form tool for evaluating the admissible inclination of rectangular sinking wells in either principal direction. The principal aim of this work is to establish a practical one-axis screening framework, while biaxial extensions and coupled admissibility domains may be developed in future studies.

Within these assumptions, the model provides a transparent closed-form tool for evaluating the admissible inclination of rectangular sinking wells in either principal direction. Its role is limited to preliminary, serviceability-level assessment of stress redistribution beneath the base. The model is not intended to replace detailed geotechnical design, non-linear FEM analysis, or field-based verification where complex soil–structure interaction, shaft flexibility, partial contact, or construction-stage effects are significant. The principal aim of this work is therefore to establish a compact one-axis screening framework, while more advanced numerical or field-calibrated analyses may be used when higher accuracy is required.

3. Results

3.1. Reference Configuration and Consistency of the Analytical Formulation

The analytical formulation presented in Section 2 was first applied to a reference rectangular sinking well in order to illustrate the stress redistribution beneath the plug caused by one-axis inclination.

The geometric parameters and load components adopted in the reference configuration are summarized in Table 1.

Table 1. Reference geometric and load parameters used in the analytical evaluation of the inclined rectangular sinking well.

Parameter	Symbol	Value
width	B	4.0 m
length	L	6.0 m
shaft height	H	8.0 m
wall thickness	t	0.25 m
plug thickness	h_c	1.0 m
concrete unit weight	γ_c	25 kN/m ³
surcharge	q_p	35 kN/m ²

Using the expressions derived in Section 2, the component loads and the resulting reference stress beneath the base were evaluated. The resulting configuration and the concept of eccentric loading associated with the inclined shaft are illustrated in Figure 1.

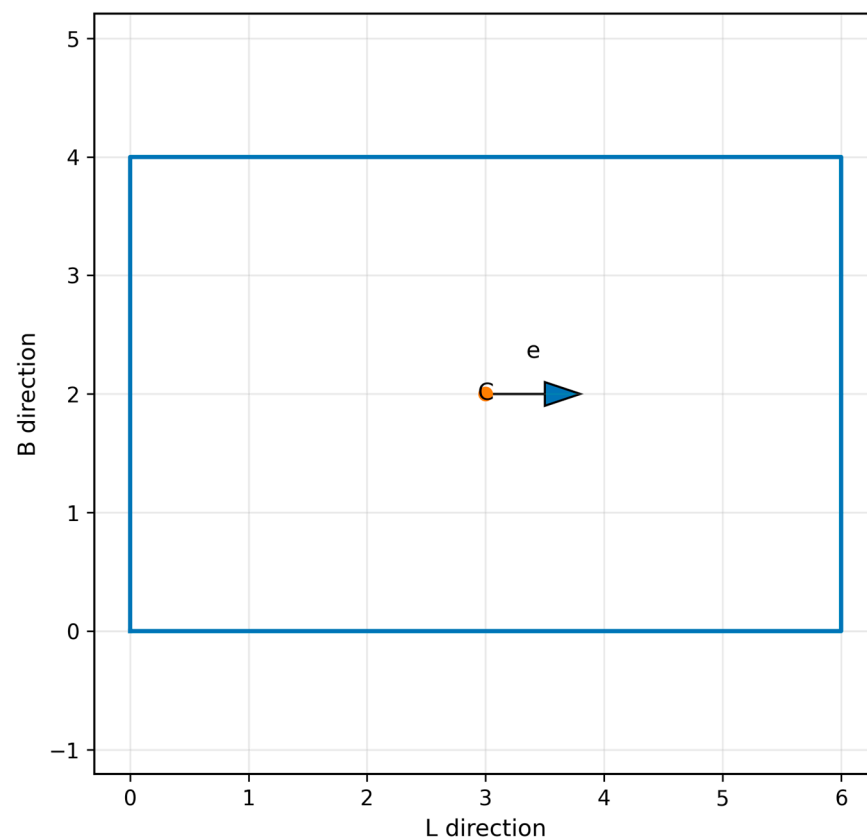


Figure 1. Plan view of the rectangular plug showing the eccentricity direction.

As shown in Figure 1, the tilt of the well produces an eccentricity of the resultant vertical load relative to the centroid of the base. This eccentricity leads to a linear redistribution

of contact stresses beneath the plug, with one edge experiencing increased compression while the opposite edge becomes partially unloaded.

For the reference case, the edge stresses obtained from Equations (22) and (23) are listed in Table 2. The stresses were computed using Equations (22) and (23) for a representative tilt angle corresponding to the admissible limit.

Table 2. Edge stresses beneath the base for the reference inclined well configuration.

Stress Location	Stress Value
maximum edge stress	$\sigma_{max} \approx 120$ kPa
minimum edge stress	$\sigma_{min} \approx 80$ kPa

The stress redistribution beneath the base is further illustrated in Figure 2, where the variation in the contact stress along the direction of tilt is plotted.

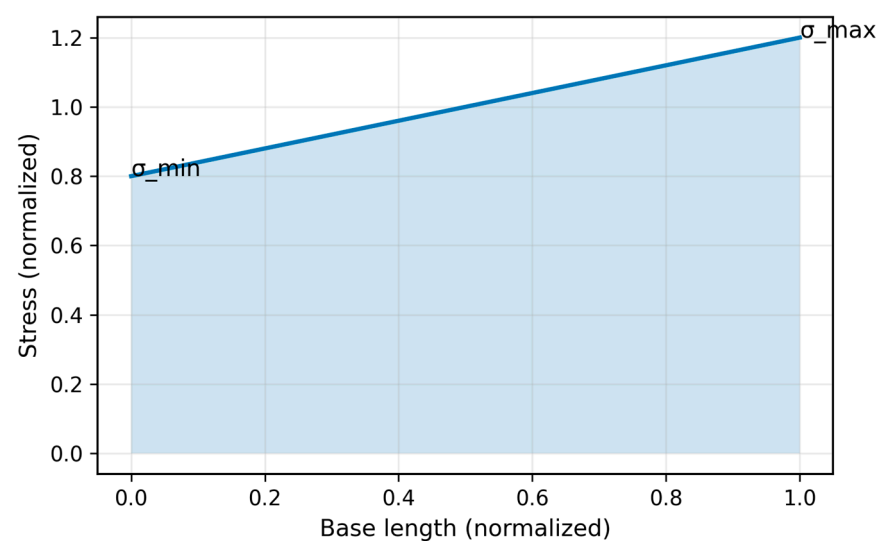


Figure 2. Linear stress distribution beneath the base of the inclined well showing the increase in stress at the loaded edge and the reduction in stress at the opposite edge.

To demonstrate the internal consistency of the proposed formulation, the predicted edge stresses were compared with the standard rigid-footing solution for eccentric vertical loading. For the reference geometry, both approaches yielded identical normalized stresses of $\sigma_{max}/\sigma_0 = 1.20$ and $\sigma_{min}/\sigma_0 = 0.80$ at the admissible state. This agreement confirms that the proposed formulation remains fully consistent with the classical analytical solution for linear stress redistribution beneath an eccentrically loaded rigid rectangular foundation.

3.2. Illustration of the Stress Amplification Criterion

Using the reference configuration, the relationship between eccentricity and stress amplification was verified by evaluating the ratio σ_{max}/σ_0 for increasing tilt angles. The resulting curve is shown in Figure 3.

As predicted by Equation (24), the stress amplification increases linearly with eccentricity and therefore increases monotonically with the tilt angle. The horizontal line corresponding to $\sigma_{max} = k_\sigma \sigma_0$ defines the admissible tilt limit according to the adopted serviceability criterion. The intersection of this line with the analytical curve yields the admissible tilt angle for the reference configuration.

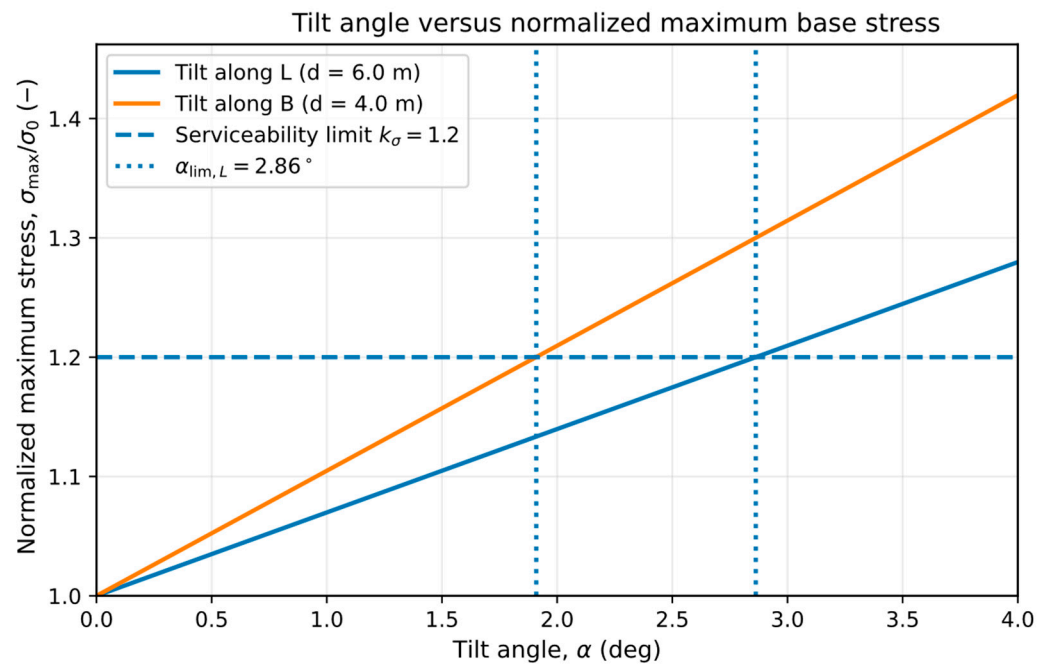


Figure 3. Relationship between the tilt angle and the normalized maximum base stress σ_{max}/σ_0 for the reference well geometry.

3.3. Influence of Shaft Height

The influence of the shaft height H on the admissible tilt angle was investigated using Equation (32). The resulting relationship between H and the admissible tilt angle is presented in Figure 4.

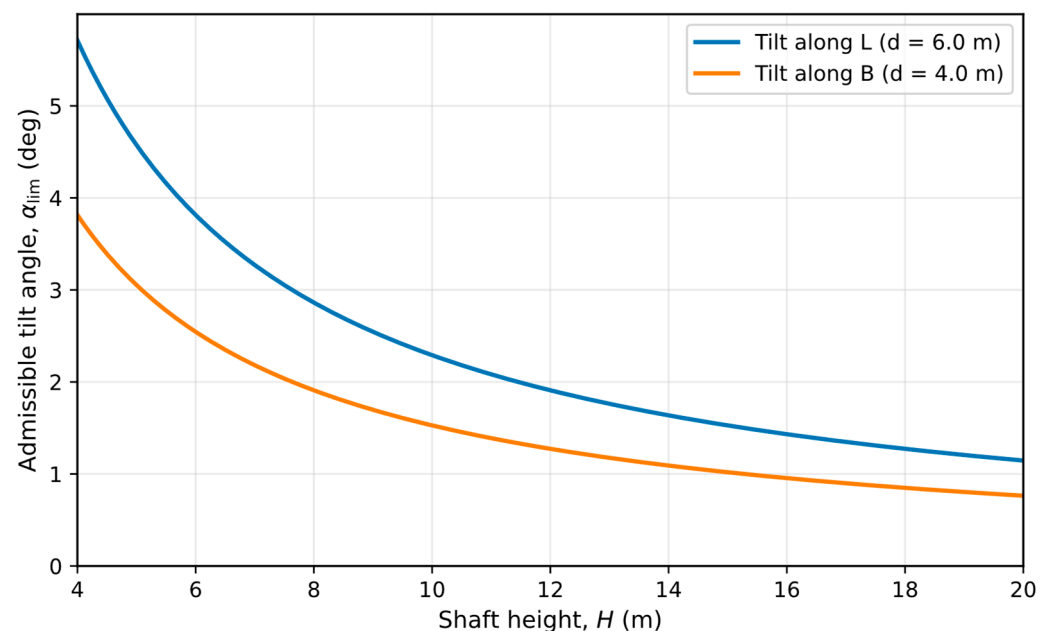


Figure 4. Influence of shaft height on the admissible tilt angle for the reference plan dimensions of the well.

As shown in Figure 4, the admissible tilt angle decreases with increasing shaft height. This behavior follows directly from the definition of eccentricity in Equation (13), where the horizontal displacement of the resultant load increases proportionally with the shaft height. Consequently, taller wells are more sensitive to small angular deviations from verticality. From a dimensionless perspective, this behavior is governed primarily by the geometric

ratio d/H , which controls the relative eccentricity generated by a given inclination angle. Wells with smaller values of d/H therefore exhibit increased sensitivity to tilt-induced stress amplification.

3.4. Influence of Plan Dimensions

The influence of the plan geometry was investigated by analyzing wells with different values of B and L while keeping the other parameters constant. The resulting admissible tilt angles in the two principal directions are summarized in Table 3.

Table 3. Admissible one-axis tilt angles for selected plan geometries of the rectangular well.

B (m)	L (m)	$\alpha_{lim,B}$ (deg)	$\alpha_{lim,L}$ (deg)
4	4	1.91	1.91
4	6	1.91	2.86
4	8	1.91	3.81
6	8	2.86	3.81

The values listed in Table 3 demonstrate that the admissible tilt angle depends strongly on the dimension of the base in the considered direction. To illustrate this effect graphically, the admissible tilt angles obtained for different plan geometries are plotted in Figure 5.

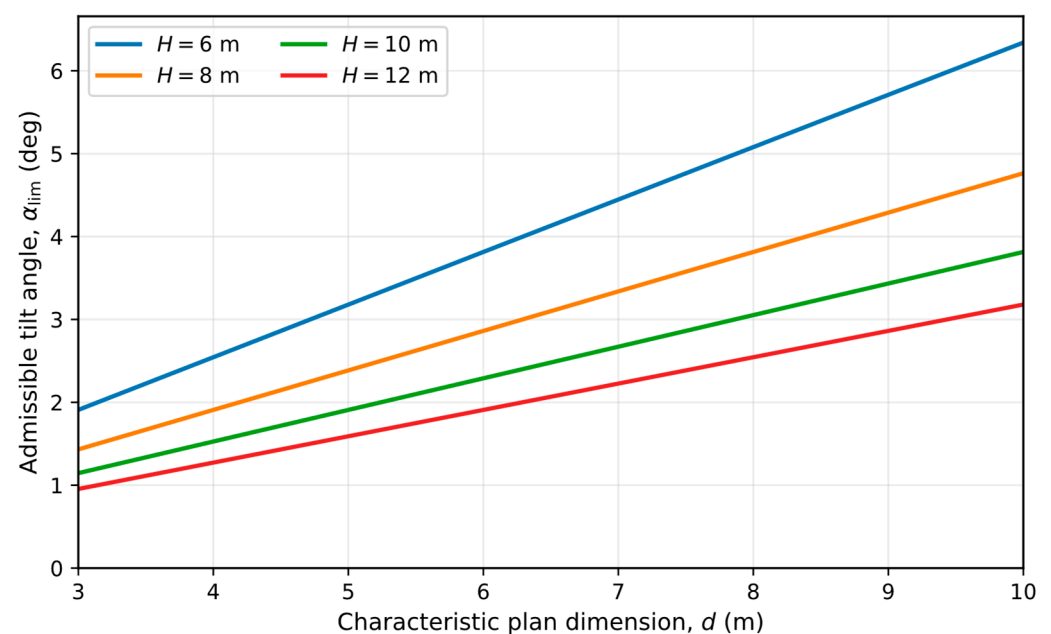


Figure 5. Influence of the plan dimensions of the well on the admissible tilt angle calculated using the analytical model.

The results presented in Figure 5 show that larger plan dimensions allow larger admissible inclinations because the eccentricity produced by a given tilt angle represents a smaller fraction of the base dimension.

3.5. Directional Anisotropy of the Admissible Tilt

For rectangular wells with unequal plan dimensions, the admissible tilt angle differs depending on the direction of inclination. This effect is illustrated in Figure 6, where the admissible tilt angles in the two principal directions are compared.

The results demonstrate that the direction corresponding to the larger plan dimension always allows a larger admissible tilt angle. This directional sensitivity reflects the

dependence of the admissible eccentricity on the characteristic plan dimension d , as shown in Equation (29). In practical terms, the same angular deviation may therefore produce significantly different stress amplification depending on the direction of inclination. This effect becomes particularly pronounced for elongated wells characterized by high plan aspect ratios.

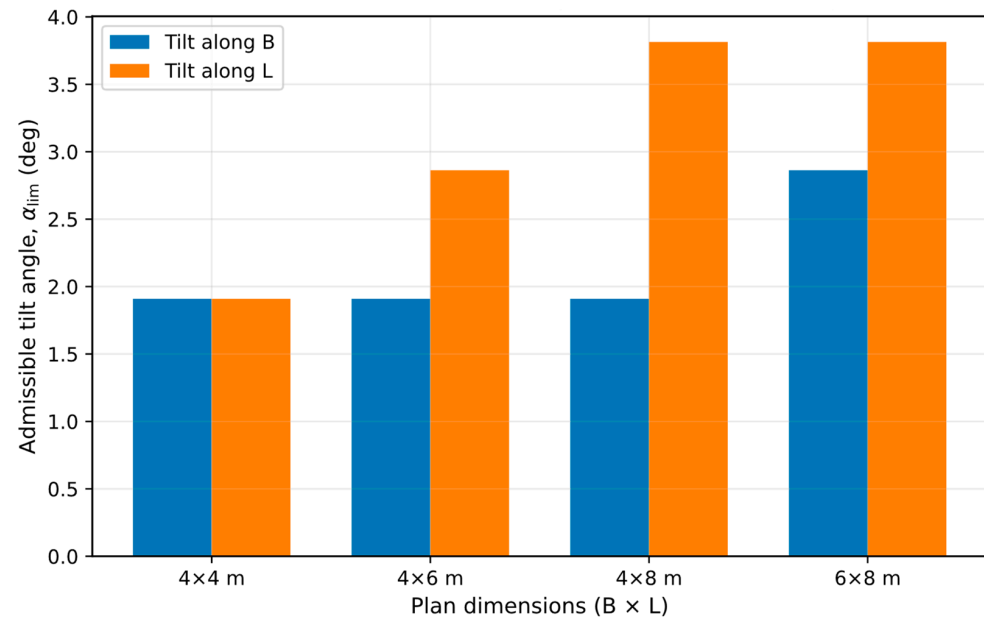


Figure 6. Comparison of admissible tilt angles for inclination in the two principal directions of the rectangular well.

3.6. Parametric Map of Admissible Tilt Angles

To provide a practical design-oriented representation of the analytical model, the admissible tilt angle was evaluated for a wide range of geometric parameters. The resulting parametric map is presented in Figure 7.

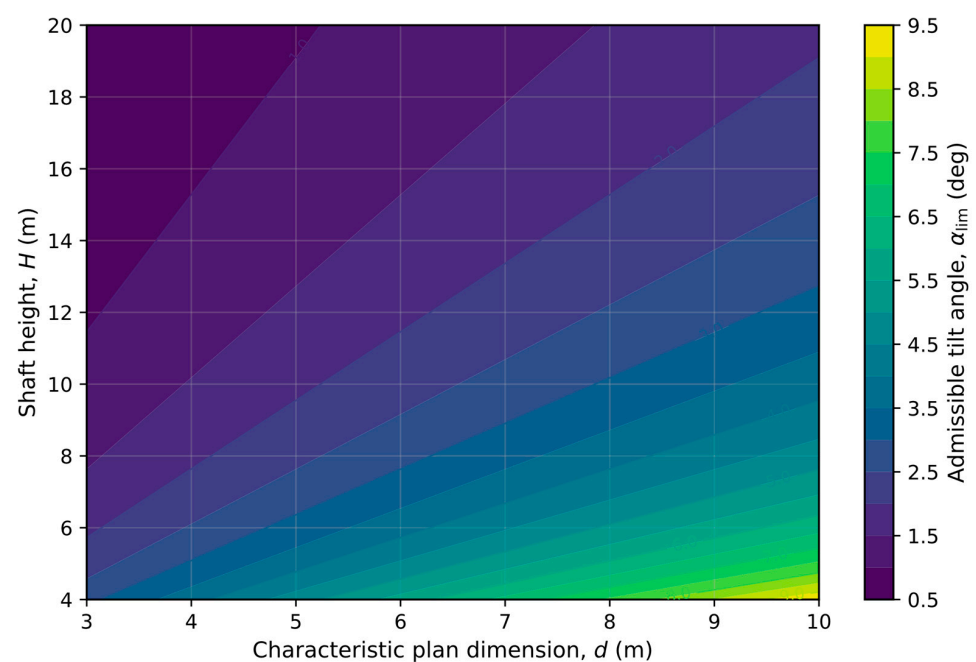


Figure 7. Parametric map of admissible tilt angles for rectangular sinking wells as a function of shaft height and plan dimension.

The chart shown in Figure 7 allows rapid estimation of the admissible tilt angle without performing additional calculations. Such charts may be useful in practice for preliminary evaluation of inclined wells during construction or post-construction inspection.

3.7. Influence of Stress Amplification Factor k_σ

The admissible tilt angle increases with increasing allowable stress amplification factor k_σ . This result indicates that project-specific serviceability tolerances may be directly incorporated into the proposed analytical framework. For conservative practice, values close to 1.2 are recommended, whereas less restrictive operational criteria may justify higher values.

3.8. Summary of the Parametric Results

The parametric study presented in Figures 3–7 and Tables 2 and 3 leads to several consistent observations.

First, the stress redistribution beneath the base of an inclined well follows a linear pattern governed by the eccentricity of the resultant vertical load. Second, the admissible tilt angle depends primarily on the ratio between the plan dimension of the base and the shaft height, which determines the magnitude of the eccentricity induced by a given angular deviation. Third, rectangular wells exhibit directional anisotropy, allowing larger inclinations in the direction of the longer plan dimension.

4. Discussion

4.1. Interpretation of the Analytical Model

The analytical framework proposed in this study provides a simple yet physically interpretable method for evaluating the admissible inclination of rectangular sinking wells. The model is based on the observation that the primary mechanical consequence of well inclination is the eccentric transfer of the vertical load to the soil through the concrete plug. This eccentricity generates a linear redistribution of contact stresses beneath the base, increasing compression at one edge while reducing it at the opposite edge.

The results presented in Figures 2 and 3 confirm that the stress distribution beneath the plug follows the classical linear pattern predicted by eccentric loading theory. The analytical expression for the normalized maximum stress, Equation (24), shows that the stress amplification depends only on the ratio between the eccentricity and the characteristic base dimension. This relationship explains why the admissible tilt angle is governed primarily by the ratio between the plan dimension of the well and its height.

In practical terms, this means that the same tilt angle may produce very different structural responses depending on the geometry of the well. For tall shafts with relatively small plan dimensions, even small angular deviations may generate significant stress amplification beneath the base. Similar linear stress redistribution under eccentric loading has been widely reported in classical foundation theory [29,34]. The present study therefore does not introduce a new stress redistribution mechanism, but rather reformulates the classical eccentric loading framework into a compact admissibility-oriented procedure applicable to inclined rectangular sinking wells.

4.2. Engineering Meaning of the Stress Amplification Criterion

A key aspect of the proposed approach is the introduction of the stress amplification criterion, Equation (26) with the reference value $k_\sigma = 1.2$. The adopted benchmark value should be interpreted as a practical engineering recommendation for serviceability-level assessment rather than as a strict codified limit. Its purpose is to define a moderate and

conservative admissibility threshold corresponding to limited stress redistribution beneath the base.

The analytical analysis presented in Section 2 shows that this limit corresponds to an eccentricity equal to only one-fifth of the classical kern limit. Consequently, the minimum contact stress beneath the plug remains positive and relatively high even at the admissible tilt limit. In fact, the normalized minimum stress at the admissible state equals, Equation (40), which indicates that the entire base remains in compression with a substantial safety margin against loss of contact.

This observation is important because it demonstrates that the proposed criterion represents a serviceability-oriented limit, rather than a structural or geotechnical ultimate limit state. In other words, the method identifies the range of tilt angles for which the stress redistribution beneath the plug remains moderate and well within the linear contact regime.

Such a conservative threshold is particularly useful in practice when evaluating the admissibility of an inclined well after construction, because it allows potentially acceptable deviations from verticality to be distinguished from cases requiring corrective measures.

4.3. Directional Sensitivity of Rectangular Wells

The parametric analysis presented in Figures 5–7 demonstrates that rectangular wells exhibit a pronounced directional sensitivity with respect to inclination.

The admissible tilt angle is proportional to the characteristic plan dimension in the considered direction, which means that the well may tolerate larger inclinations in the direction of the longer side of the rectangle. For square wells, where $B = L$, the admissible tilt angle is identical in both directions, whereas elongated wells exhibit clear anisotropy.

From a practical standpoint, this observation has important implications for construction monitoring and post-construction assessment. When evaluating the inclination of an existing well, the direction of the tilt should therefore be considered explicitly rather than relying on a single scalar measure of deviation from verticality.

4.4. Limitations of the Proposed Model

The analytical model developed in this study is intentionally simplified in order to provide a transparent engineering tool. As such, several limitations should be recognized.

First, the model assumes that the well behaves as a rigid body and that the concrete plug remains rigidly connected to the shaft. In reality, some flexibility of the structure and the soil may lead to deviations from the ideal linear stress distribution assumed in the analysis. Such deviations may become more pronounced for very slender wells, highly deformable soils, or situations involving nonlinear settlement response and partial loss of contact beneath the base. Under such conditions, more advanced soil–structure interaction analyses may be required.

Second, the soil beneath the plug is represented implicitly through the assumption of compressive contact without explicit modeling of soil deformation or non-linear constitutive behavior. Therefore, the proposed method should be interpreted as a screening-level analytical tool rather than a substitute for detailed geotechnical analysis.

Third, the present formulation considers only one-axis inclination at a time. While this assumption is appropriate for many practical situations where the well tilts predominantly in a single direction, more complex cases involving combined inclinations may require a more general formulation.

In practical sinking wells, shaft skin friction may transfer part of the self-weight directly to surrounding soil, thereby reducing the effective vertical force acting on the base. Typical engineering estimates range from 20% to 40% depending on soil type and construction technology. This effect may be incorporated by replacing N with ξN , where

$\xi = 0.6\text{--}0.8$. Since the present model uses the full load N , the predicted admissible tilt remains conservative. It should be noted, however, that shaft friction may also influence the effective location of the resultant vertical force, not only its magnitude. The simplified reduction-factor approach adopted here does not explicitly model this redistribution effect and should therefore be interpreted only as a preliminary engineering approximation.

4.5. Implications for Practical Assessment of Inclined Wells

Despite its simplicity, the proposed analytical framework offers several advantages for practical engineering assessment.

First, the method provides a closed-form expression for the admissible tilt angle that depends only on basic geometric parameters of the well. This allows rapid preliminary evaluation of the structural implications of measured deviations from verticality. For very tall and slender wells, even relatively small inclination angles may correspond to substantial horizontal displacements at the top of the shaft. Such effects may influence operational serviceability, alignment of connected installations, or equipment functionality, even if the base stresses remain within the admissible range considered in the present study.

Second, the analytical structure of the model makes it possible to generate parametric charts, such as the maps shown in Figure 7, which can serve as convenient design or diagnostic tools.

Finally, the model clearly separates the effects of geometry, load eccentricity and stress redistribution, thereby providing a transparent physical interpretation of the response of inclined wells.

The simplicity of the formulation is intentional and represents a key advantage for rapid engineering assessment.

4.6. Relation to Extended Biaxial Formulations

The present study focuses on the case of one-axis inclination, which represents the most common and practically relevant scenario in many construction situations.

However, in cases where the well encounters localized obstacles or experiences asymmetric resistance during sinking, the resulting tilt may occur simultaneously in two orthogonal directions. In such situations, the stress redistribution beneath the plug becomes two-dimensional and the extreme stresses occur at the corners rather than along the edges of the base.

A more general analytical framework capable of describing such biaxial inclination scenarios is therefore required for a complete assessment of the admissibility of tilted wells. The extension of the present model to biaxial tilt and the corresponding combined admissibility domain is addressed in the companion study. For preliminary engineering use, biaxial inclination may be conservatively checked by verifying each projected component independently:

$$\alpha_x \leq \alpha_{\text{lim},B}, \alpha_y \leq \alpha_{\text{lim},L}$$

This projection rule defines a simple rectangular admissibility domain. The proposed projection rule should nevertheless be interpreted only as a simplified preliminary approximation. Under true biaxial eccentricity, the maximum contact stresses may develop at the corners of the base and may therefore exceed the stresses predicted using separate one-axis checks considered independently.

5. Conclusions

This study presents a simple analytical framework for evaluating the admissible inclination of rectangular sinking wells based on the redistribution of contact stresses beneath the concrete plug. The proposed model focuses on the case of single-axis inclination,

which represents the most common practical scenario encountered during construction and post-construction assessment of sinking wells.

Based on the analytical derivations and the parametric analyses presented in this work, the following conclusions can be drawn:

1. Inclination of a sinking well leads to eccentric transfer of the vertical load, which generates a linear redistribution of contact stresses beneath the base of the concrete plug. The maximum compressive stress develops at one edge of the base, while the opposite edge experiences a reduction in stress.
2. The stress amplification beneath the base depends only on the ratio between the eccentricity and the characteristic plan dimension of the well. This relationship leads to a compact analytical expression for the normalized maximum stress (Equation (24)).
3. The admissible tilt angle can be expressed in a closed analytical form that depends solely on the plan dimension of the well and the shaft height (Equation (32)). This expression provides a transparent engineering tool for preliminary evaluation of inclined wells.
4. The adopted stress amplification criterion $k_\sigma = 1.2$ corresponds to a conservative serviceability-based limit. At this level of stress increase, the minimum contact stress beneath the plug remains approximately $0.8\sigma_0$, which ensures that the entire base remains in compression and that loss of contact with the soil does not occur.
5. The admissible inclination depends strongly on the geometric proportions of the well. Wells with larger plan dimensions tolerate larger tilt angles, whereas tall wells with relatively small bases are significantly more sensitive to angular deviations from verticality.
6. Rectangular wells exhibit directional anisotropy of the admissible tilt angle. Inclination in the direction of the longer plan dimension results in a smaller relative eccentricity and therefore allows larger admissible tilt angles than inclination in the direction of the shorter side.
7. The proposed analytical framework offers a transparent screening tool for practical engineering assessment. Because the model depends only on basic geometric parameters, it allows rapid estimation of admissible tilt angles during construction monitoring or post-construction inspection of sinking wells.
8. The proposed framework should be interpreted primarily as a transparent analytical screening tool intended for preliminary engineering assessment under moderate inclination and serviceability-level conditions. More advanced analyses may be required in cases involving strong soil nonlinearity, significant shaft flexibility, biaxial interaction effects, or complex construction-stage phenomena.
9. Square wells represent the symmetry-preserving special case of the formulation, exhibiting identical admissible tilt limits in both orthogonal directions, whereas rectangular wells show geometry-induced directional anisotropy.

The present work addresses the case of one-axis inclination, which provides the simplest analytical description of the problem. In practical situations, wells may also experience simultaneous inclinations in two orthogonal directions, leading to two-dimensional stress redistribution beneath the base. The present framework therefore provides a rational basis for future extensions toward biaxial admissibility models.

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