Influence of Geometric Parameters on Internal Forces in the Walls of Rectangular Tanks

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Abstract

Rectangular tanks are commonly used in various industries for storing materials and products. The design of reinforced concrete liquid tanks, which must be preceded by a static analysis, is a complex issue requiring specialized knowledge and engineering experience. All types of actions, design situations, and resulting load combinations must be considered, including deformations caused by temperature gradients and the interaction of the bottom plate with the ground. Most tanks are designed and constructed with constant wall thickness, regardless of their rectangular or circular cross-section. However, tanks with variable wall thickness (e.g., trapezoidal cross-section) are rarely designed, despite their optimal fit to stress distribution. For hydrostatically loaded tanks, the load on walls increases with depth, causing the highest bending moments at the wall-bottom connection, while the value at the top, free edge is zero. Thus, structural and economic considerations favour walls with thickness increasing with depth. This article presents the results of a verification of static calculations of a monolithic rectangular tank with trapezoidal cross-section walls, comparing it with three other commonly designed tanks with different thickness and wall designs. Static calculations were performed using the finite difference method in terms of energy, employing the condition for the minimum energy of elastic strain stored in a bent plate resting on the elastic base. Traditional calculation methods were used by discretizing the object and creating systems of equations. Analysis of the results shows that constructing walls of linearly variable thickness results in a redistribution of bending moments compared to tanks with uniform wall thickness. This significantly impacts the required reinforcement area. Tanks with linearly variable wall thickness are more economical in terms of material use, aligning with the principles of sustainable construction.

Keywords: *rectangular tanks, finite difference method, trapezoidal cross-section walls, variable walls thickness*

1. Introduction

Proper water management has become a necessity in the modern world. Severe weather phenomena including droughts and floods are affecting more and more areas, both rural and urbanized. In each of these places appropriate water management activities should be carried out [1,2]. Tanks are construction objects widely used to store water and other products resulting from technological processes. Taking into consideration the material from which tanks are made, we can distinguish steel, concrete and plastic tanks. The typification of these structures applies mainly to steel and plastic tanks, less frequently to concrete ones. This is due to the fact that they constitute a single shipping element, i.e. there is no need to assemble such an object at the place of installation because it is ready for build-in immediately after arrival. There has been a huge development in the design of typical steel tanks for water [3], gas [4] or bulk materials [5], making it easier for potential users, particularly in case of small capacities, to purchase a tank and install it without having to obtain administrative decisions. This increases the possibilities of proper storage of water, grain, waste and other products related to specific business activities. Considering plastic or composite tanks, their application mainly includes small facilities for collecting waste or storing water [6]. The typification of reinforced concrete tanks applies primarily to objects with small capacities, with larger objects it is always difficult to make tight joints-locks between their components [7]. However, in many cases, reinforced concrete tanks are designed individually in accordance with the guidelines and needs of the investor and then the user [8], taking into account designed strength parameters and stored products, which may have a destructive effect on concrete. However, this is not a reason to exclude concrete as a construction material for producing tanks. Currently, technologies used to protect concrete surfaces from the harmful effects of materials stored in tanks are being advanced at a rapid pace [9,10].

Correct design of tanks requires knowledge of their statics and mutual dependencies between their elements. By using traditional solutions, i.e. by dividing a rectangular tank into individual plates as in the separated plate method, calculations concern separate plates: walls, the bottom and the cover. Considering the correctness of results, if the difference in moments is not large, i.e. up to 10%, the higher value is considered reliable. Whereas, when the difference in results is greater, the so-called equalisation of moments is used, known as the Cross method. It consists in dividing the difference in moments from the supporting plates converging at the edge of the tank into the plates proportionally to their rigidity [7,8].

Tanks are most often designed with walls of constant thickness, yet the use of trapezoidal cross-section walls is optimal in terms of utilizing their load-bearing capacity. This undoubtedly makes sense in all structures where load distribution is triangular, i.e. when walls are loaded with hydrostatic pressure. Since the load increases with an increase in depth, wall thickness should also accordingly increase with depth. The advantage of using tanks with walls of variable thickness is the economic aspect resulting from reduced material use, whereas the disadvantage lies in the difficulties associated with their production [11]. There are many literature articles referring to tanks with constant wall thickness [12,13], which describe the principles of correct design as well as possible errors and corrective methods [7,8]. Loads acting on tanks can be grouped as permanent actions, i.e. dead weight and backfill soil for underground tanks, or as variable environmental and operational actions, i.e. snow load, vehicle backfill, soil resistance and soil friction against the wall for tanks constructed using the technology of sunk wells. Less frequently described in the literature and less recognized by designers is the effect of temperature. Temperature can work on building objects in two ways: by uniform heating or cooling of the entire cross-section of the element or by occurrence of the temperature difference between planes of the element [14]. Thinner walls in the upper part of a tank, which is usually more exposed to thermal effects, are justified when we take into account that bending moments caused by the temperature difference ΔT between individual wall planes increase in direct proportion to the square of their thickness. The literature on the subject includes significantly fewer scientific works on tanks with walls of linearly variable thickness, particularly those exposed to the influence of temperature changes [15,16]. These issues were addressed, among others, in the works [11,17], where a numerical analysis of the structure of a plate of convergent thickness subjected to thermal loads was performed [17] and numerical calculations for a tank with walls of variable thickness subjected to experimental verification were presented [11].

The aim of this work is to compare the values of deflections and bending moments for four monolithic rectangular tanks of identical dimensions, differing only in the thickness and structure of walls. Calculations were made for tanks standing on the ground, filled with liquid and subject to thermal loads. The essence of the work is to indicate the benefits of using trapezoidal cross-section walls and the risks caused by their possible replacement by walls of constant thickness. All this bearing in mind that the optimisation process in construction is a very broad and challenging issue. Hence, it seems advisable to promote construction solutions that provide optimal use of load-bearing capacity in relation to acting loads.

2. Materials and Methods

Static calculations of tanks can be made using several popular numerical methods. These include, for example, the boundary element method, the finite difference method, and the finite element method. Computational programs for dimensioning structures such as tanks are most often based on the finite element method. In this work, the finite difference method was used to solve the designated systems of differential equations as an alternative and equally effective one. It is a very universal way of solving differential equations under certain boundary conditions, consisting in replacing the derivatives present in the equation and boundary conditions by appropriate difference ratios. Since the function describing the deflection of the plate is unknown, the ordinates of deflection in a finite number of points called nodes located at the points of intersection of the assumed division mesh of the calculated structure are assumed as unknown [18]. The subject of the finite difference method was taken up in many outstanding and fundamental scientific works in the seventies and eighties of the last century [19-24] constituting a contribution and inspiration for the works of contemporary authors [25-29]. FDM has been used in numerical calculations of plates [17,29,30], tanks [11-13] and surface girders.

This work used the condition for the minimum energy of elastic deformation accumulated while undergoing bending in the plate resting on the elastic base. Calculations were performed traditionally, discretizing the object and creating systems of equations. Then, using proprietary calculation solutions, the results were obtained, i.e. deflections at each point of the division mesh and the values of bending moments at selected points. The function describing the elastic deformation energy and potential energy was described by Formula (1) [18].

$$V = \frac{D}{2}\iint_A \left\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2v\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] + 2(1+v)\frac{\alpha_t \Delta T}{h}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\alpha_t \Delta T}{h}\right)\right\} dA + \frac{1}{2}\iint_A K w^2 dA - \iint_A q w dA \qquad (1)$$

Where:

$D = \frac{Eh^3}{12(1-v^2)}$ – plate flexural rigidity,

ν – Poisson’s ratio,

h – plate thickness,

w – plate deflection,

q – load perpendicular to the central surface of the plate, ΔT – difference in temperature between the lower plate T_d and the upper plate T_g determined by correlation: $\Delta T = T_d - T_g$,

α_t – coefficient of thermal expansion of the plate material,

K – subgrade stiffness reaction,

A – plate area.

Calculations were performed for Poisson's ratio ν=0. The analysis assumed a tank with linearly variable wall thickness of h_0 in the upper part and h_{10} in the lower part (Fig. 1).

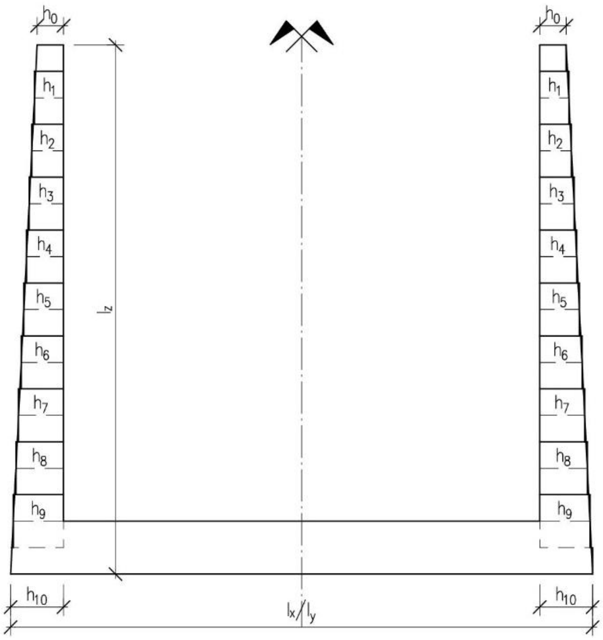

Fig. 1. Cross-sectional view of the tank with marked mesh division for the wall with linearly variable thickness.

For the assumed discretization grid, being ten meshes along the plate height, the values of wall thickness in the individual meshes of the division grid were calculated using general formulas. For this purpose, the constant λ was taken, which specifies the rigidity ratio of the bottom part to the upper part of the plate. The tank under consideration took λ=8. The dependencies of individual thicknesses on the thickness h_0 are presented below.

The constant λ is defined by the following relations:

$$\frac{D_{10}}{D_0} = \lambda, \quad \frac{h_{10}^3}{h_0^3} = \lambda, \quad \frac{h_{10}}{h_0} = 2 \tag{2}$$

where:
h_{10} – maximum plate thickness,
h_0 – minimum plate thickness,
D_{10} – plate rigidity with h_{10},
D_0 – plate rigidity with h_0,
After transforming the dependence of the constant λ (2) the result is:

$$h_{10} = h_0\sqrt[3]{\lambda}, \quad D_{10} = \lambda\, D_0 \tag{3}$$

The following results were obtained for the following plate thicknesses:

$$h_9 = h_0 + \frac{9}{10}(h_{10} - h_0) = h_0\left[1 + \frac{9}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_9 = D_0\frac{h_9^3}{h_0^3} \tag{4}$$

$$h_8 = h_0 + \frac{8}{10}(h_{10} - h_0) = h_0\left[1 + \frac{8}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_8 = D_0\frac{h_8^3}{h_0^3} \tag{5}$$

$$h_7 = h_0 + \frac{7}{10}(h_{10} - h_0) = h_0\left[1 + \frac{7}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_7 = D_0\frac{h_7^3}{h_0^3} \tag{6}$$

$$h_6 = h_0 + \frac{6}{10}(h_{10} - h_0) = h_0\left[1 + \frac{6}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_6 = D_0\frac{h_6^3}{h_0^3} \tag{7}$$

$$h_5 = h_0 + \frac{5}{10}(h_{10} - h_0) = h_0\left[1 + \frac{5}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_5 = D_0\frac{h_5^3}{h_0^3} \tag{8}$$

$$h_4 = h_0 + \frac{4}{10}(h_{10} - h_0) = h_0\left[1 + \frac{4}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_4 = D_0\frac{h_4^3}{h_0^3} \tag{9}$$

$$h_3 = h_0 + \frac{3}{10}(h_{10} - h_0) = h_0\left[1 + \frac{3}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_3 = D_0\frac{h_3^3}{h_0^3} \tag{10}$$

$$h_2 = h_0 + \frac{2}{10}(h_{10} - h_0) = h_0\left[1 + \frac{2}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_2 = D_0\frac{h_2^3}{h_0^3} \tag{11}$$

$$h_1 = h_0 + \frac{1}{10}(h_{10} - h_0) = h_0\left[1 + \frac{1}{10}\left(\sqrt[3]{\lambda} - 1\right)\right], \quad D_1 = D_0\frac{h_1^3}{h_0^3} \tag{12}$$

For the point where the tank wall thickness changes, as shown in Fig. 1, the rigidity value Ds was assumed as an arithmetic mean, i.e.:

$$D_{s01} = 0.5\,(D_0 + D_1) \tag{13}$$

After solving the system of displacement equations, there were obtained the values of bending moments in all nodes of the assumed discretization grid of the tank with linearly variable wall thickness. Based on deflections, there were calculated the values of bending moments according to the following Formulas:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\alpha_t \Delta T}{h}\right) \tag{14}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \frac{\alpha_t \Delta T}{h}\right) \tag{15}$$

The results obtained for the tank with linearly variable wall thickness were compared with the results for tanks with constant and stepwise variable wall thicknesses [31].

3. Results

For numerical calculations, four identical tanks with axial dimensions: lx = 10 m, ly = 5 m, lz = 5 m, standing on the ground, with constant thickness of the bottom of 0.5 m, differing only in the thickness and structure of walls were assumed. Tank no. 1 had walls of constant thickness of 0.25 m, tank no. 2 had walls with stepwise variable thickness, i.e. in the lower part up to half the height 0.5 m, in the upper part 0.25 m. For tank no. 3, walls were assumed with constant thickness equal to the thickness of the bottom, i.e. 0.5 m, while tank no. 4 had walls with linearly variable thickness of 0.25 m at the top and 0.5 m at the bottom.

The analysed case concerned the plate with λ = 8, which means that the lower part of the plate was eight times more rigid in relation to its upper part. When the thickness of a plate is considered, this corresponds to the case where the lower part is twice as thick as the upper part and is equal to the thickness of the bottom of the tank. According to Formulas (4) to (12), the following was assumed:

$h_0 = 1$ and $D_0 = 1$,
$h_1 = 1.1\ h_0$ and $D_1 = 1.331\ D_0$,
$h_2 = 1.2\ h_0$ and $D_2 = 1.728\ D_0$,
$h_3 = 1.3\ h_0$ and $D_3 = 2.197\ D_0$,
$h_4 = 1.4\ h_0$ and $D_4 = 2.744\ D_0$,
$h_5 = 1.5\ h_0$ and $D_5 = 3.375\ D_0$,
$h_6 = 1.6\ h_0$ and $D_6 = 4.096\ D_0$,
$h_7 = 1.7\ h_0$ and $D_7 = 4,.913\ D_0$,
$h_8 = 1.8\ h_0$ and $D_8 = 5.832\ D_0$,
$h_9 = 1.9\ h_0$ and $D_9 = 6.859\ D_0$,
$h_{10} = 2\ h_0$ and $D_{10} = 8\ D_0$.

It was assumed that the tanks rested on a soil substrate described by the Winkler model with compliance modulus of K=50,000 kN/m3, elasticity modulus of the wall material E=30 GPa, Poisson's ratio v=0 and the coefficient of linear thermal expansion α_t = 1 · 10-5/°C.

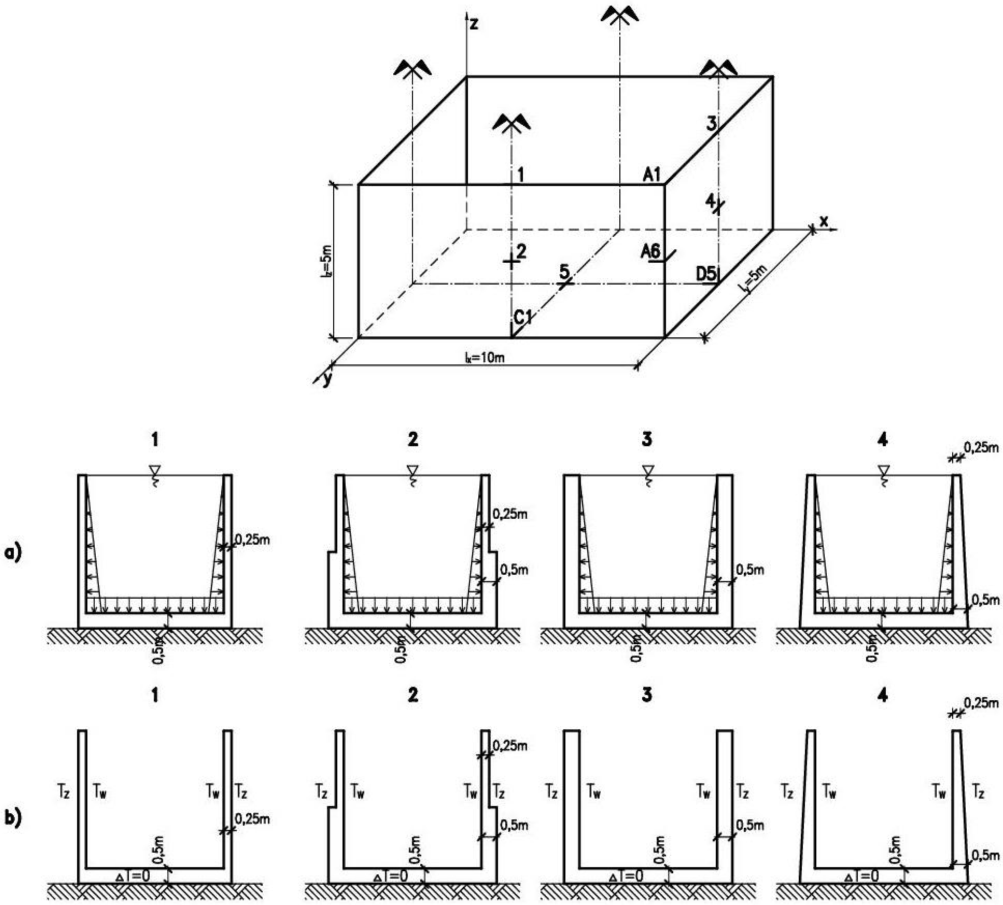

Fig. 2. Basic denominations, characteristic cross-sections and load diagrams adopted for calculations: a) hydrostatic load, b) thermal load. Tanks 1,2,3 according to [31], tank no. 4 according to this work.

The results of calculations are presented in Tables 1 and 2, providing the values of deflections and bending moments at selected points of the tank shown in Fig. 2. Table 1 presents the values for hydrostatic load on walls and the bottom, while Table 2 presents the load as the temperature difference between the planes of tank walls.

The analysed tanks were subjected to the following actions:

- uniform loads acting on the bottom due to hydrostatic pressure (q_1 = 50 kN/m^2),
- hydrostatic pressure acting on walls, with the tank completely filled (q_2 = 50 kN/m^2),
- thermal load acting on walls due to the temperature difference ΔT, with the assumed values T_z = -5 °C, T_w = 15°C, $\Delta T = T_z - T_w$ = -20 °C, and ΔT = 0 for the bottom.

Calculations for tanks no. 1, 2, 3 [31] were compared with calculations performed in a traditional way using the finite difference method in variational approach (1) for the tank with linearly variable wall thickness. In calculations in areas with abruptly changing thickness, the value of rigidity was assumed as an arithmetic mean. When assuming a square division grid with a mesh size of s = 0.5 m, for a quarter of the tank, there were obtained for determining deflections in 200 nodes of the division grid. Based on the obtained deflections, bending moments were calculated at characteristic points of the tank.

Fig. 1 shows basic denominations, characteristic cross-sections, and load diagrams.

Tab. 1. Summary for tanks of different thickness and wall structure. The load is the pressure of liquid on walls and the bottom. Tanks 1,2,3 according to [31], tank no. 4 according to this work

Denomination acc. to Fig. 1	**Unit**	**Tank no. 1**	**Tank no. 2**	**Tank no. 3**	**Tank no. 4**
w_1	cm	0.730	0.233	0.097	0.198
w_2		0.412	0.102	0.056	0.087
w_3		-0.073	0.003	-0.011	-0.005
w_4		0.010	0.009	-0.001	0.001
w_5		0.016	0.011	0.018	0.015
M_{x1}	kNm/m	33.899	10.125	36.029	12.038
M_{A1}		-50.884	-24.241	-52.535	-27.135
M_{y3}		-0.744	4.630	-2.509	2.126
M_{x2}		17.513	18.664	19.486	19.023
M_{A6}		-47.318	-65.666	-47.894	-43.528
M_{y4}		8.609	14.040	6.531	12.233
M_{z1}		0	0	0	0
M_{z2}		16.987	-12.718	18.564	7.129
M_{C1}		-112.524	-125.280	-102.719	-107.452
M_{y5}		38.576	34.480	39.573	36.289
M_{z3}		0	0	0	0
M_{z4}		8.097	-1.339	7.727	5.985
M_{D5}		-30.647	-40.750	-40.633	-40.689
M_{x5}		-1.090	-2.040	0.801	-1.982

Tab. 2. Summary for tanks of different thickness and wall structure. The load is the temperature difference between the planes of tank walls. Tanks 1,2,3 according to [31], tank no. 4 according to this work

Denomination acc. to Fig. 1	**Unit**	**Tank no. 1**	**Tank no. 2**	**Tank no. 3**	**Tank no. 4**
w_1	cm	0.259	0.300	0.098	0.289
w_2		0.018	0.026	-0.020	0.009
w_3		0.027	0.061	0.019	0.011
w_4		-0.024	-0.005	-0.013	-0.008
w_5		0.014	0.024	0.030	0.026
M_{x1}	kNm/m	40.884	41.763	150.777	48.369
M_{A1}		-25.883	-37.578	-93.402	-39,254
M_{y3}		37.366	44.975	153.563	47.235
M_{x2}		32.814	99.886	121.098	103.841
M_{A6}		37.358	83.334	172.955	128.156
M_{y4}		27.288	89.508	103.424	92.187
M_{z1}		0	0	0	0
M_{z2}		18.101	17.227	61.333	19.287
M_{C1}		27.246	47.025	67.804	51.214
M_{y5}		10.680	18.700	22.311	19.874
M_{z3}		0	0	0	0
M_{z4}		29.277	50.822	118.977	65.218
M_{D5}		29.461	57.300	46.650	51.187
M_{x5}		0.366	0.500	3.804	0.852

By analysing the results collected in Tables 1 and 2, it can be stated that the thickness and structure of tank walls are important for the values of obtained bending moments. In the case of designing such tanks, this has a decisive effect on the assumed required reinforcement area. On the other hand, changing the structure of walls from linearly variable thickness to constant thickness can cause a failure or even a catastrophe of a given tank, because the values of bending moments increase significantly. Walls with variable thickness, both linearly and abruptly variable, are more optimal for the acting load, the value of which increases with the depth of the tank. Therefore, it is justified to assume thinner walls at the top and thicker at the bottom.

4. Discussion

Financial and socio-economic benefits are the factors that determine investment management in all countries. Strategies, planning documents and legal regulations concerning, among others, hydrotechnical facilities used for water management are published and announced in very general terms [32].

The authors analysed four types of tanks differing in the thickness and structure of walls in order to select the most economically advantageous solution and therefore the most appropriate and optimal one in terms of material consumption. A tank with linearly variable wall thickness is the most advantageous design solution. Calculations were made using the finite difference method, which is equivalent to the finite element method in terms of the obtained results. Considering the number and variety of structures or structural elements calculated using the finite element method [33-36], it can be assumed that the finite difference method is less popular among the authors of scientific papers, therefore the authors of this article wanted to present this method based on their earlier works in this field [11-13,17,30,31]. Tanks with linearly or stepwise variably thick walls made as steel structures are discussed in [37], which concerns cylindrical tanks. In this paper ''the weighted smeared wall method'' was introduced, which is described as simpler than previous methods of transforming the tank first into a three-stage cylinder and then into an equivalent cylinder of uniform thickness. However, this two-stage process led to complicated calculations which are difficult in practical design of silos and tanks. In the presented weighted smeared wall method, calculations made traditionally, much more accessible for the designer using a spreadsheet, were compared with accurate finite element calculations using ABAQUS, and obtained a good convergence of results. The aim of the work [38] was to investigate the effect of variable wall thickness of cylindrical steel tanks open at the top on the buckling of the tank wall under the influence of settlement. The study was carried out on four tanks that had the same geometric and material properties except for wall thickness. The conclusions from the cited work can also be applied to reinforced concrete tanks because the authors state that the use of walls of constant thickness improves the buckling stability of steel walls. Yet, it comes with an increase in costs. When simplifying the analysis and omitting variable wall thickness, it is important to take the value of thickness as the arithmetic mean of the thickness over the step change of wall thickness, similarly as in this work. The work [39] considers the method of calculating thermal stresses in cylindrical reinforced concrete tanks with variable wall thickness, fixed at the base in the bottom plate and subjected to axisymmetric uniform thermal load. Similarly to the rectangular tanks considered in this work, thermal stresses originating only from circumferential forces in the tank shell can significantly exceed the average value of tensile strength of the considered concrete. Therefore, thermal shrinkage of the shell not designed for thermal load can cause its serious cracking. The work [40] includes the analysis of a reinforced concrete cylindrical tank with variable wall thickness subjected to hydrostatic load and thermal load. The bottom plate in this case supported on an elastic half-space can be bent due to uniformly distributed load, whereas when the Winkler foundation is adopted, it is bent only by edge forces, which results in opposite values of moments at the connection of the plate and the cylindrical shell. It is contrary to the assumed temperature gradient, because then, over the entire thickness of the shell, the distribution of internal forces in the tank and horizontal displacements of the tank wall have opposite signs. The article [17] presents the results of static calculations performed using the finite difference method for rectangular plates with linearly variable thickness, which have a trapezoidal cross-section, with three fixed edges and one free edge subjected to constant hydrostatic and thermal loads. In addition to the numerical analysis, the article also presents the results of model tests of a plate with linearly variable thickness made of resin loaded with temperature. The convergence of the obtained results proves the correctness of the performed calculations and tests and is a significant contribution to the recognition of the statics of rectangular plates with a trapezoidal cross-section. The paper [11] presents the results of verification of static calculations of a monolithic rectangular tank with trapezoidal cross-section walls, performed using a computer program based on the finite element method and the finite difference method in the energy approach. The verification of the obtained results was carried out on a tank model made of concrete using a modern coordinate measuring tool of a measuring arm with a contact head. According to the paper [11], it can be assumed that the systems of equations for a tank with linearly variable thick walls constructed in accordance with the principles of the finite difference method are correct and the obtained results of model tests confirm their correctness. In this paper, the authors used the same method, and their own spreadsheets based on the finite difference method.

5. Conclusions

By analysing the obtained solutions concerning the walls of tanks, it can be stated that when constructing walls with linearly variable thickness, the values of bending moments are rearranged in relation to walls of equal thickness.

Furthermore:

- for the tank with linearly variable wall thickness, there was a significant reduction in bending moments at the upper edge, for hydrostatic load acting on walls and uniform load acting on the bottom of the tank,
- for thermal load, there was a very large reduction in bending moments in all cross-sections of the tank with linearly variable wall thickness compared to the tank with constant 50 cm wall thickness,
- bending moments at the upper edge of the tank with linearly variable wall thickness, caused by the temperature difference ΔT, are greater than in the tank with walls 25 cm thick,
- maximum bending moments caused by the temperature difference ΔT for the tank with linearly variable wall thickness are greater than could be obtained from the Formula $M_t = \frac{Eh^2}{12}\alpha_t \Delta T$, which is often used in practice when determining bending moments in plate structures or tanks loaded with temperature,
- bending moments from the acting thermal load increase in direct proportion to the square of wall thickness, therefore, referring to the analysed tank with walls of variable thickness, it can be stated that the value of the moment for the upper part of the plate with thickness h_0 is $M_t = \frac{Eh_0^2}{12}\alpha_t \Delta T$, while for the lower part of the plate, where $h_{10}=2h_0$ it is equal to $M_t = 4\frac{Eh_0^2}{12}\alpha_t \Delta T$.

In engineering practice, the most common are tanks designed and constructed with walls of constant thickness due to the ease of execution compared to walls with linearly variable thickness. Yet, the most desirable solution for tanks are walls of linearly variable thickness, adapted to the values of bending moments that reach their highest value in the lower part, and the upper one on the free edge takes the zero value. The values of bending moments presented in Tables 1 and 2 due to hydrostatic load and thermal load show how the use of walls of variable thickness is an optimal design solution. The above statements lead to the conclusion that structural and economic considerations should determine the selection of walls with thickness increasing accordingly to the depth of the tank since the material consumption for such walls is lower, thus more cost-effective. However, a huge danger and possible failure or catastrophe of the tank is posed by the inexpert abandonment – most often at the stage of tank construction – of walls of variable thickness in favour of walls of constant, greater thickness, because then the values of bending moments caused by thermal load increase in direct proportion to the square of wall thickness.

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